Forecasting with Temporal Hierarchies

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Outline

1. Motivation
2. Temporal Hierarchies
3. Do they work? Empirical results
4. Significance for practice – Predicting A&E
5. Cross-temporal hierarchies
   & decision making
Decision making and forecasting

Decision making in organisations has at its core an element of forecasting:

- Accurate forecasts lead to reduced uncertainty → better decisions
- Forecasts maybe implicit or explicit

Forecasts aims to provide information about the future, conditional on historical and current knowledge.

Company targets and plans aim to provide direction towards a desirable future.

Difference between targets and forecasts, at different horizons, provide useful feedback.
Forecasts are central in decisions relating to:

- Inventory management
- Promotional and marketing activities
- Logistics
- Human resource planning
- Purchasing and procurement
- Cash flow management
- Building new production/storage unit
- Entering new markets
- ...

Accurate forecasts can support
- Decision making
- Identifying and capitalising on opportunities
- Cost saving
Decisions need to be aligned:

- Operational short-term decisions
- Tactical medium-term decisions
- Strategic long-term decisions

Shorter term plans are **bottom-up** and based mainly on **statistical forecasts** & expert adjustments.

Longer term plans are **top-down** and based mainly on **managerial expertise** factoring in unstructured information and organisational environment.

Given different sources of information (and views) forecasts will differ → plans and decisions not aligned.

**Objective**: construct a framework to reconcile forecasts of different levels and eventually align decisions → less waste & costs, agility to take advantage of opportunities.
The Idea

• Given some time series data we can temporally aggregate it

• Temporal aggregation strengthens and attenuates different elements of the series:
  - at an aggregate level trend/cycle is easy to distinguish
  - at a disaggregate level high frequency elements like seasonality typically dominate.

• Modelling a time series at a very disaggregate level (e.g. weekly) → short-term forecast. The opposite is true for aggregate levels (e.g. annual)

• Propose Temporal Hierarchies that provide a framework to optimally combine information from various levels (irrespective of forecasting method) to i) reconcile forecasts; ii) avoid over-reliance on a single planning level.
Temporal Hierarchies build on two ideas:

1. The Multiple Aggregation Prediction Algorithm (MAPA) [Kourentzes et al., 2014]

2. Optimal combination for cross-section hierarchical forecasts [Hyndman et al., 2011; Athanasopoulos et al., 2009]
Motivation - MAPA

Multiple Aggregation Prediction Algorithm

**Step 1:** Aggregation

- $Y^{[1]}$  
  - $k = 2$  
    - $Y^{[2]}$  
    - $k = 3$  
      - $Y^{[3]}$  
      - $k = K$  
        - $Y^{[K]}$

**Step 2:** Forecasting

- ETS Model Selection
- $l^{[i]}$  
- $b^{[i]}$  
- $s^{[i]}$

**Step 3:** Combination

- $K^{-1} \sum (\text{Strengthens and attenuates components})$
- $\hat{Y}^{[1]}$
- $\hat{Y}^{[2]}$
- $\hat{Y}^{[3]}$
- $\hat{Y}^{[K]}$

Estimation of parameters at multiple levels

Robustness on model selection and parameterisation

Strengthens and attenuates components
Motivation - MAPA

The good bits:

- Very good at picking up long term dynamics → aggregation filters high frequency elements and strengthens low frequency ones;
- Lessens importance of model selection/parameter estimation → forecast combination;
- Removes the need to select a single aggregation level;
- Increases accuracy over base forecasts.

The bad bits:

- Described for exponential smoothing (ETS), not readily usable for other forecasting methods. Does not incorporate human judgement;
- Ad-hoc combination weights → evidence that better weights must exist.
In many forecasting applications there are multiple time series that are hierarchically organised:

- Product categories
- Market segments
- Geographical
- Supply channels
- etc.

There are two major advantages in considering hierarchical structure in forecasting:

- Improve accuracy
- Reconcile forecasts → Decision making advantages
A typical example of hierarchical forecasting comes from cross-sectional applications. For example let us suppose we want to forecast student admissions:

Enrolled students

UK/EU
Management Science
Maths & Stats

International
Management Science
Maths & Stats

Reconciling forecasts across levels helps improve accuracy and align decision making.
**Motivation - Hierarchical Forecasting**

**Top-down** hierarchical methods:

- At the top level forecasting should be easier (noise is reduced)
- But details from the lower levels are lost
- Single level dominates

![Diagram showing the hierarchy of enrolled students: Enrolled students at the top, followed by UK/EU and International, then Management Science and Maths & Stats.](image-url)
**Motivation - Hierarchical Forecasting**

**Top-down** hierarchical methods:

- Bottom level details are retained
- But typically very noisy
- Single level dominates
Motivation - Hierarchical Forecasting

Optimal combination

- “Optimally” combine forecasts from all levels to produce reconciled cross-sectional forecasts
- No single level dominates → weights derived from hierarchy structure
Analogously we can construct temporal hierarchies, where now we consider for a single time series multiple levels of temporal aggregation.

The idea is that similarly to cross-sectional hierarchies, we can take advantage of the structure to increase accuracy at short and long term and align forecasts of different horizons.
Temporal Hierarchies - Notation

Non-overlapping temporal aggregation to $k^{th}$ level:

$$y_j^{[k]} = \sum_{t=t^*+(j-1)k}^{jk} y_t,$$

Observations at each aggregation level
Collecting the observations from the different levels in a column:

\[ y_i = \left( y_i^{[m]}, \ldots, y_i^{[k_3]}, y_i^{[k_2]}, y_i^{[1]} \right) \]

We can define a “summing” matrix \( S \) so that:

\[ y_i = S y_i^{[1]} \]

\( S = \)

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{array}
\]

- Lowest level observations
- Annual
- Semi-annual
- Quarter
Example: Monthly

Aggregation levels $k$ are selected so that we do not get fractional seasonalities.
We can arrange the forecasts from each level in a similar fashion:

\[ \hat{y}_h = (\hat{y}_h^{[m]}, \ldots, \hat{y}_h^{[k_3]'}, \hat{y}_h^{[k_2]'}, \hat{y}_h^{[1]'})' \]

The reconciliation model is:

\[ \hat{y}_h = S\beta(h) + \varepsilon_h \]

The reconciliation error has zero mean and covariance matrix \( \Sigma \).
If $\Sigma$ was known then we can write (GLS estimator):

$$\tilde{y}_h = S\hat{\beta}(h) = S(S'\Sigma^{-1}S)^{-1}S'\Sigma^{-1}\hat{y}_h = SP\hat{y}_h$$

But in general it is not known, so we need to estimate it.

It can be shown that $\Sigma$ is not identifiable (you need to know the reconciled forecasts, before you reconcile them), however:

$$\text{Var}(y_{T+h} - \tilde{y}_h) = SPWP'S'$$

So our problem becomes:

$$\tilde{y}_h = S(S'W^{-1}S)^{-1}S'W^{-1}\hat{y}_h$$

Reconciliation errors

Covariance of forecast errors
All we need now is an estimation of $W$

$$\Lambda = \frac{1}{\left\lfloor \frac{T}{m} \right\rfloor} \sum_{i=1}^{\left\lfloor \frac{T}{m} \right\rfloor} e_i e'_i$$

Sample covariance of in-sample errors

In principle this is fine, but its sample size is controlled by the number of top-level (annual) observations. For example 104 observations at weekly level, results in just 2 sample points (2 years).

So the estimation of $\Lambda$ is typically weak in practice.
Temporal Hierarchies - Forecasting

We propose three ways to estimate it, with increasing simplifying assumptions.

Using as example quarterly data the approximations are:

**Hierarchy variance scaling**

\[ \Lambda_H = \text{diag}\left(\hat{\sigma}_A^4, \hat{\sigma}_{SA_1}^2, \hat{\sigma}_{SA_2}^2, \hat{\sigma}_{Q_1}^1, \hat{\sigma}_{Q_2}^1, \hat{\sigma}_{Q_3}^1, \hat{\sigma}_{Q_4}^1\right)^2 \]

**Series variance scaling**

\[ \Lambda_V = \text{diag}\left(\hat{\sigma}_A^4, \hat{\sigma}_{SA_1}^2, \hat{\sigma}_{SA_2}^2, \hat{\sigma}_{Q_1}^1, \hat{\sigma}_{Q_2}^1, \hat{\sigma}_{Q_3}^1, \hat{\sigma}_{Q_4}^1\right)^2 \]

**Structural scaling**

\[ \Lambda_S = \text{diag}\left(4, 2, 2, 1, 1, 1, 1\right) \]

**Diagonal of covariance matrix** → less elements to estimate

- Assume within level equal variances. This is what conventional forecasting does. Increases sample size.
- Assume proportional error variances. No need for estimates → can be used when unknown (e.g. expert forecasts).
The forecast (blue line) is the result of reconciling the forecasts across all temporal aggregation levels. Here ETS was used, but any model can be used and it does not have to be the same across levels.
Evaluation on the M3 dataset

<table>
<thead>
<tr>
<th>Aggregation level</th>
<th>h</th>
<th>ETS Base</th>
<th>ETS BU</th>
<th>ETS WLS_H</th>
<th>ETS WLS_V</th>
<th>ETS WLS_S</th>
<th>ARIMA Base</th>
<th>ARIMA BU</th>
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RMAE

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<th>ETS WLS_H</th>
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MASE

**BU**: Bottom-Up; **WLS_H**: Hierarchy scaling; **WLS_V**: Variance scaling; **WLS_S**: Structural scaling

1453 series, forecast t+1 - t+18 months ahead
**Evaluation on the M3 dataset**

**Quarterly dataset**

<table>
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<th>Aggregation</th>
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</table>

**RMAE**

| Annual               | 1     | -14.6    | -15.8    | -15.9    | -17.2  | 1.6      | -20.6    | -22.1    | -22.1    | -19.7    |
| Semi-annual          | 4     | -6.8     | -7.8     | -7.9     | -9.1   | 1.2      | -2.9     | -4.7     | -4.5     | -1.6     |
| Quarterly            | 8     | 0.0      | -0.6     | -1.1     | -2.6   | 1.2      | 0.0      | -1.6     | -1.4     | 1.5      |
|                      | Average| -7.1     | -8.1     | -8.3     | -9.6   | -7.8     | -9.5     | -9.3     | -6.6     |

**MASE**

**BU**: Bottom-Up; **WLS$_H$**: Hierarchy scaling; **WLS$_V$**: Variance scaling; **WLS$_S$**: Structural scaling

756 series, forecast t+1 - t+8 quarters ahead
Evaluation on the M3 dataset

Comparison with other M3 results (symmetric Mean Absolute Percentage Error):

- **Monthly dataset**
  - **Temporal**: 13.61% (WLS<sub>S</sub>)
  - **ETS**: 14.45% [Hyndman et al., 2002]
  - **MAPA**: 13.89% [Kourentzes et al., 2014]
  - **Weighted MAPA**: 13.69%
  - **Theta**: 13.85% (best original performance) [Makridakis & Hibon, 2000]

- **Quarterly dataset**
  - **Temporal**: 9.70% (WLS<sub>V</sub>)
  - **ETS**: 9.94% [Hyndman et al., 2002]
  - **MAPA**: 10.18% [Kourentzes et al., 2014]
  - **Weighted MAPA**: 9.58%
  - **Theta**: 8.96% (best original performance) [Makridakis & Hibon, 2000]
What do Temporal Hierarchies do?

• Temporal aggregation filters information:
  – Multiple views of the series;
  – When forecasts reconciled $\rightarrow$ equivalent to shrinkage [Current work]

• Forecast combinations:
  – Reduce forecast error variance;
  – Reduce model uncertainty in terms of specification & estimation $\rightarrow$
    supported by simulations.

• Independent of forecasting model:
  – Any base forecasting method can be used;
  – Does not need to be the same across levels;
  – Forecasts may be produced/adjusted by human experts.

• Reconciled forecasts:
  – Forecasts agree across different forecast horizons/objectives.
A major challenge in running A&E wards is predicting the staffing requirements.

**Objectives:**

- Staffing schedules (up to 4 weeks ahead)
- Staff planning and procurement (3 months ahead)
- Staff training (1 year ahead)
- Calibrate existing plans (1 week)

**Why?**

- In the UK A&E departments need to service 95% of patients within 4 hours.
- This is difficult to achieve due to staffing problems, shortages, bed capacity, etc.
An application: Predicting A&E admissions

Collect weekly data for UK A&E wards.
13 time series: covering different types of emergencies and different severities (measured as time to treatment)
Span from week 45 2010 (7th Nov 2010) to week 24 2015 (7th June 2015)
Series are at England level (not local authorities).

Accurately predict to support staffing and training decisions.
Aligning the short and long term forecasts is important for consistency of planning and budgeting.

Test set: 52 weeks.
Rolling origin evaluation.
Forecast horizons of interest: t+1, t+4, t+52 (1 week, 1 month, 1 year).
Evaluation MASE (relative to base model)
As a base model auto.arima (forecast package R) is used.
An application: Predicting A&E admissions

Total Emergency Admissions via A&E

Red is the prediction of the base model (ARIMA)
Blue is the temporal hierarchy reconciled forecasts (based on ARIMA)

Observe how information is `borrowed’ between temporal levels. Base models for instance provide very poor weekly and annual forecasts.
**An application: Predicting A&E admissions**

<table>
<thead>
<tr>
<th>Aggr. Level</th>
<th>h</th>
<th>Base</th>
<th>Reconciled</th>
<th>Change</th>
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<tbody>
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<td>3.4</td>
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</table>

- Accuracy gains at all planning horizons
- Crucially, forecasts are reconciled leading to aligned plans
Hierarchical (or grouped) forecasting can improve accuracy, but their true strength lies in the reconciliation of the forecasts → aligning forecasts is crucial for decision making.

Is the reconciliation achieved useful for decision making?

### Cross-sectional
- Reconcile across different items.
- Units may change at different levels of hierarchy.
- Suppose an electricity demand hierarchy: lower and higher levels have same units. All levels relevant for decision making.
- Suppose a supply chain hierarchy. Weekly sales of SKU are useful. Weekly sales of organisation are not! Needed at different time scale.

### Temporal
- Reconcile across time units/horizons.
- Units of items do not change.
- Consider our application. NHS admissions short and long term are useful for decision making.
- Suppose a supply chain hierarchy. Weekly sales of SKU is useful for operations. Yearly sales of a single SKU may be useful, but often not!
- Operational → Tactical → Strategic forecasts.
Temporal hierarchies permit aligning operational, tactical and strategic planning, while offering accuracy gains → useful for decision making

BUT there can be cases that strategic level forecasts are not required for each item, but at an aggregate level.

Let us consider tourism demand for Australia as an example. Local authorities can make use of detailed forecasts (temporal/spatial) but at a country level weekly forecasts are of limited use.

- Temporal: tactical → strategic
- Cross-sectional: local → country

Cross temporal can support decisions at both dimensions:
- Tactical/local; • strategic/local; • tactical/country; • strategic/country
Cross-temporal hierarchies: Tourism demand

56 (bottom level) quarterly tourism demand series → Athanasopoulos et al. (2011)

- 6 years in-sample
- 3 years out-of-sample horizon: up to 2 years
- rolling origin evaluation

Cross-temporal hierarchical forecasts:
- Most accurate
- Most complete reconciliation (one number forecast)
- Flexible decision making support

<table>
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<th>MAPE %</th>
<th>Level</th>
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<th>Theta</th>
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<td>28.74</td>
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<tr>
<td></td>
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<td>4</td>
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<td></td>
<td>Bottom</td>
<td>56</td>
<td>36.32</td>
<td>32.58</td>
</tr>
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</table>

Temporally reconciled

|        | Overall  | 89            | 30.46   | 28.19  |
|        | Top      | 1             | 5.75    | 6.13   |
|        | Level 1  | 4             | 9.29    | 9.04   |
|        | Level 2  | 28            | 27.18   | 24.21  |
|        | Bottom   | 56            | 34.06   | 31.95  |

Cross-temporally reconciled

|        | Overall  | 89            | 30.26   | 28.04  |
|        | Top      | 1             | 6.02    | 5.88   |
|        | Level 1  | 4             | 9.11    | 8.70   |
|        | Level 2  | 28            | 25.91   | 23.87  |
|        | Bottom   | 56            | 34.39   | 31.90  |
Conclusions

• Temporal hierarchies provide a new class of hierarchical forecasts that can be produced for any time series.

• Applicable to forecasts produced by any means → theoretically elegant hierarchical combination of forecasts.

• Joins operational, tactical and strategic decision making by reconciling forecasts → satisfies a business need that has remained unmet

• Potential to increase forecasting accuracy and mitigate modelling uncertainty

• Combining cross-sectional and temporal hierarchies: forecasts reconciled across conventional hierarchy and forecast horizons → one number forecast → superior decision making.
Can these be used?

• **Multiple Aggregation Prediction Algorithm (MAPA)**
  – **R package on CRAN: MAPA**
  – All papers, code and examples available on my website ([http://nikolaos.kourentzes.com](http://nikolaos.kourentzes.com))

• **Hierarchical (cross-sectional) forecasting**
  – **R package on CRAN: hts**

• **Temporal Hierarchies** → Working paper and R code soon on my website!
Thank you for your attention!

Questions?

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