MONASH University

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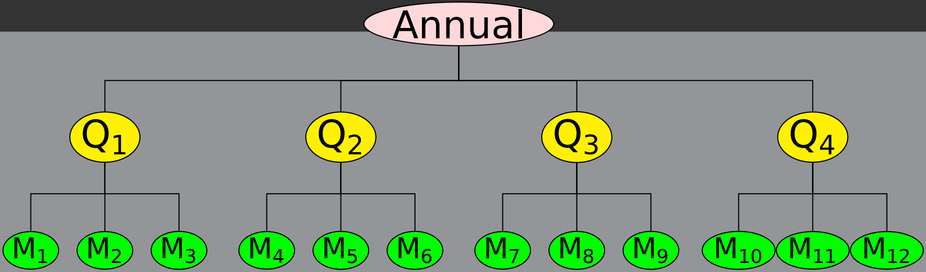
(with Rob J. Hyndman, Nikos Kourentzes and Fotis Petropoulos)

Forecasting with temporal hierarchies



Outline

1 Introduction

2 Temporal hierarchies

3 Optimal combination forecasts

4 A Monte-Carlo simulation study

5 Conclusion

Temporal aggregation

Key issue:

➥ Aggregating model/forecasts versus modelling/forecasting the aggregate.

- Temporal aggregation literature: Amemiya and Wu (1972), Tiao (1972), Brewer (1973), Wei (1978, 1980), Rosanna and Seater (1992, 1995),..., Silvestrini et al. (2008), Silvestrini and Veredas (2008).
- All within the ARIMA framework.

Temporal aggregation

1. Effect on the structure of dynamics.
   - Aggregation complicates/contaminates/changes dynamics.
   - Loss of information of components.
2. Parameter estimation efficiency.
   - Losses always happen here no matter what model you are considering.
3. Effect on forecasting. What is the optimal level of aggregation?
   - Results vary both empirically and in simulations.
   - Impossible to set some guidelines for the empirical analyst/forecaster.
   - Can use disaggregate series to forecast aggregates but not visa versa.

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Basic idea

Kourentzes, Petropoulos, Trapero (2014), MAPA, *IJF*.

- For a series observed at the highest possible frequency, construct aggregate series up to the annual level.
- Do not choose a level of aggregation. Forecast all series and optimally combine resulting in a set of reconciled forecasts at all frequencies.

Key implication:

- Reconciled forecasts align managerial objectives. How do we do in forecast accuracy?

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General notation

We set aggregation levels k to be a factor of m, the highest sampling frequency per year. E.g., for quarterly series, $m = 4$, we consider three levels of aggregation: $k = \{4, 2, 1\}$.

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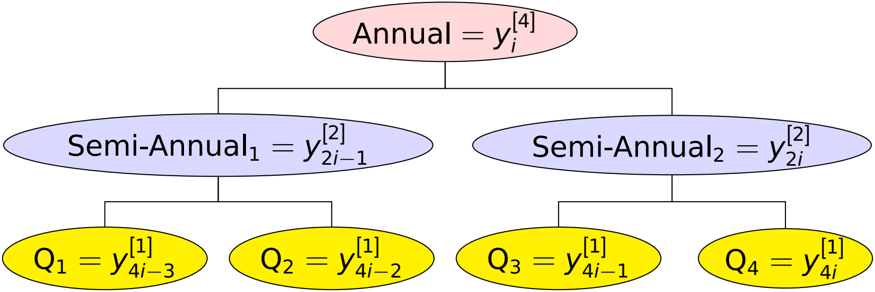

General notation

Collecting these in one column vector,

$$\boldsymbol{y}_i = \left(y_i^{[4]}, \boldsymbol{y}_i^{[2]'}, \boldsymbol{y}_i^{[1]'}\right)'.$$

Hence,

$$\boldsymbol{y}_i = \boldsymbol{S}\boldsymbol{y}_i^{[1]}.$$

For $m = 4$,

$$\boldsymbol{S} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ & \boldsymbol{I}_4 & & \end{bmatrix}$$

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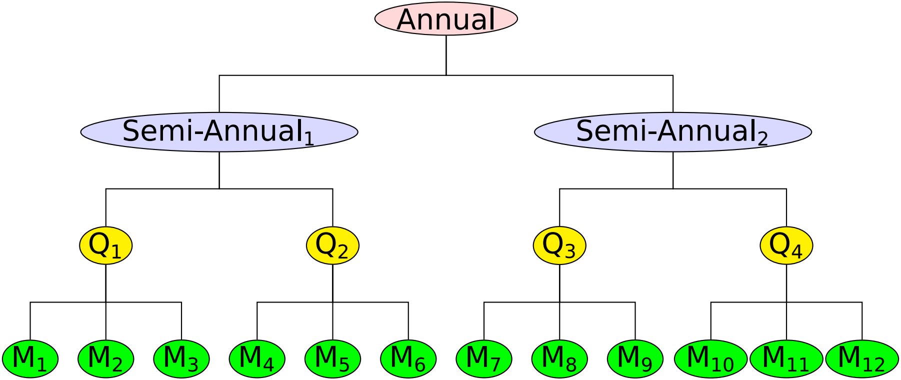
General notation: monthly data



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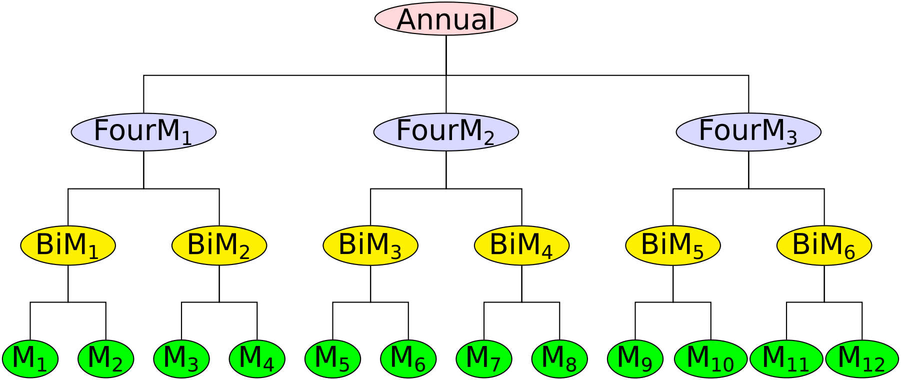

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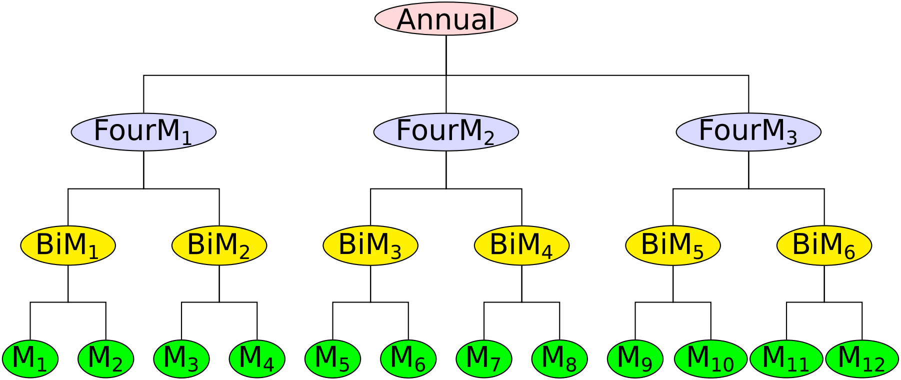

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$$
\underbrace{\begin{pmatrix} A \\ SemiA_1 \\ SemiA_2 \\ FourM_1 \\ FourM_2 \\ FourM_3 \\ Q_1 \\ \vdots \\ Q_4 \\ BiM_1 \\ \vdots \\ BiM_6 \\ M_1 \\ \vdots \\ M_{12} \end{pmatrix}}_{\boldsymbol{y}_i}
=
\underbrace{\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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 & & & & & & \vdots & & & & & \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
 & & & & & & \boldsymbol{I}_{12} & & & & &
\end{pmatrix}}_{\boldsymbol{S}}
\underbrace{\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \\ M_{10} \\ M_{11} \\ M_{12} \end{pmatrix}}_{\boldsymbol{y}_i^{[1]}}
$$

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3 **Optimal combination forecasts**

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Forecasting framework

Let h be the required forecast horizon at the annual level. For each aggregation level k we generate $m/k \times h$ base forecasts and stack them the same way as the data,

$$\hat{\boldsymbol{y}}_h = (\hat{y}_h^{[m]}, \dots, \hat{\boldsymbol{y}}_h^{[k_3]'}, \hat{\boldsymbol{y}}_h^{[k_2]'}, \hat{\boldsymbol{y}}_h^{[1]'})'.$$

Reconciliation regression,

$$\hat{\boldsymbol{y}}_h = \boldsymbol{S}\beta(h) + \varepsilon_h$$

where $\beta(h) = E[\boldsymbol{y}_{\lfloor T/m \rfloor + h}^{[1]} | y_1, \dots, y_T]$ and ε_h is the reconciliation error with mean zero and covariance Σ_h.

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Approx. optimal forecasts

$$\tilde{\boldsymbol{y}}_h = \boldsymbol{S}\hat{\beta}(h) = \boldsymbol{S}(\boldsymbol{S}'\Sigma_h^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\Sigma_h^{-1}\hat{\boldsymbol{y}}_h$$

Solution 1: OLS

- Approximate Σ_h by $\sigma^2\boldsymbol{I}$.

Solution 2: WLS (variance scaling)

- Let $\Lambda = \left[\text{diagonal}\left(\hat{\Sigma}_1\right)\right]$ contain the one-step forecast error variances.

$$\tilde{\boldsymbol{Y}}_h = \boldsymbol{S}(\boldsymbol{S}'\Lambda^{-1}\boldsymbol{S})^{-1}\boldsymbol{S}'\Lambda^{-1}\hat{\boldsymbol{Y}}_h$$

- Easy to estimate, and places more weight where we have best forecasts.

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Solution 3: WLS (structural scaling)

- Bottom level reconciliation errors have approximately the same variances.
- Assuming that they are approximately uncorrelated then Σ_h is proportional to the number of series contributing to each node.
- So set $\Sigma_h = \sigma^2\Lambda$ where

$$\Lambda = diag(\boldsymbol{S} \times \mathbf{1})$$

where $\mathbf{1} = (1, 1, \ldots, 1)'$.

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Simulation setup

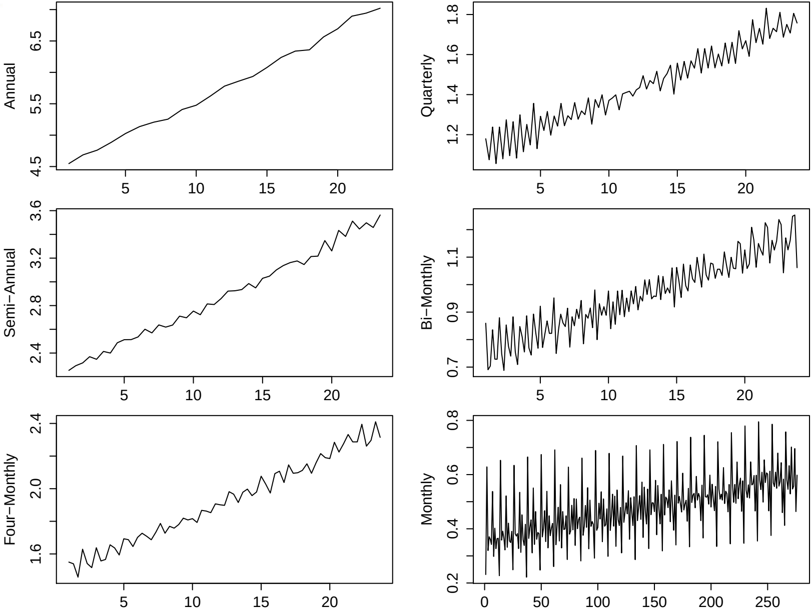
- Silvestrini and Veredas (2008, *JoES*): Survey paper on temporal aggregation.
- Theoretical derivation of temporarily aggregated ARIMA models.
- Empirical application Belgian cash deficit series, 252 monthly observations, ARIMA(0,0,1)(0,1,1)$_{12}$ with an intercept.
- Discussion on estimation efficiency loss.
- Two simulation setups.

Simulation setup 1

Freq	ARIMA orders	$\hat{c}$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\Theta}_1$	$\hat{\sigma}_e$
		Theoretically derived parameters				
Annual	(0,1,2)	112.3	-0.43	0.01		
SemAnn	$(0,0,1)(0,1,1)_2$	28.1	-0.05		-0.4	
FourM	$(0,0,1)(0,1,1)_3$	12.4	-0.06		-0.4	
Quart	$(0,0,1)(0,1,1)_4$	7.0	-0.10		-0.4	
BiMonth	$(0,0,1)(0,1,1)_6$	3.1	-0.13		-0.4	
		$\times 10^3$				
		Estimated parameters				
Monthly	$(0,0,1)(0,1,1)_{12}$	0.78	-0.22		-0.4	4.19
		$\times 10^3$				$\times 10^{-5}$

Drawing from $\varepsilon_t \sim N(0, \hat{\sigma}^2_\varepsilon)$, we generate time series from the monthly DGP and then aggregate these to the levels above.

ARIMA(0,0,1)(0,1,1)$_{12}$ with drift



Simulation setup 1

Four scenarios. Base forecasts for each series at each aggregation level generated from:

1. the theoretically derived ARIMA DGPs at each level (complete certainty);
2. the theoretically derived correct ARIMA specification but with estimated parameters (parameter uncertainty);
3. an automatically selected ARIMA model (model uncertainty);
4. an automatically selected ETS model (partial model misspecification).

Simulation setup 1

Forecast comparisons

1. Approx. optimal combination using WLS (Variance).
2. Bottom up.

versus base (unreconciled) forecasts.

Negative (positive) entries represent a percentage decrease (increase) in RMSE compared to base (unreconciled) forecasts.

Iterations

- Sample sizes (annual): 4, 12, 20, 40.
- Forecast horizons (annual): 1, 3, 5, 10.
- 1,000 iterations for each sample size.

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Simulation setup 1: results

Scenario 1 (complete certainty):
Base forecasts from theoretically derived ARIMA DGPs.

Sample (h)	4 (1)	12 (3)	20 (5)	40 (10)	4 (1)	12 (3)	20 (5)	40 (10)
	WLS				**Bottom-up**			
Annual	-0.3	0.0	0.0	0.0	-0.7	-0.1	0.2	0.1
SemiA	-0.2	-0.1	0.0	0.0	-0.5	-0.1	0.1	0.0
FourM	-0.1	0.0	0.0	0.0	-0.2	-0.1	0.1	0.0
Quart	-0.1	0.0	0.0	0.0	-0.2	0.0	0.0	0.0
BiM	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0
Monthly	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

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FourM	-0.1	0.0	0.0	0.0	-0.2	-0.1	0.1	0.0
Quart	-0.1	0.0	0.0	0.0	-0.2	0.0	0.0	0.0
BiM	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0
Monthly	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Negative (positive) entries represent a percentage decrease (increase) in RMSE compared to base (initial) forecasts.

Simulation setup 1: results

Scenario 2 (parameter uncertainty):
Base forecasts from estimated ARIMA DGPs at each level.

Sample	4	12	20	40	4	12	20	40
(h)	*(1)*	*(3)*	*(5)*	*(10)*	*(1)*	*(3)*	*(5)*	*(10)*
	WLS				**Bottom-up**			
Annual	-4.3	-7.9	-6.1	-3.3	-5.3	-9.5	-7.1	-3.4
SemiA	-5.2	-3.5	-1.6	-0.2	-7.6	-4.8	-2.4	-0.2
FourM	-3.7	-1.4	-0.4	-0.1	-5.5	-2.6	-0.9	-0.2
Quart	-3.9	-0.6	-0.2	-0.1	-6.0	-1.8	-0.7	-0.2
BiM	-1.1	0.0	0.1	0.0	-2.8	-0.9	-0.2	-0.1
Monthly	1.0	0.4	0.1	0.0	0.0	0.0	0.0	0.0

Negative (positive) entries represent a percentage decrease (increase) in RMSE compared to base (initial) forecasts.

Simulation setup 1: results

Scenario 3 (model uncertainty):
Base forecasts from automatically selected ARIMA models.

Sample	4	12	20	40	4	12	20	40
(h)	*(1)*	*(3)*	*(5)*	*(10)*	*(1)*	*(3)*	*(5)*	*(10)*
	WLS				**Bottom-up**			
Annual	-66.2	-5.1	-2.6	-0.4	-64.2	-1.2	5.9	27.9
SemiA	-50.6	-4.9	-2.6	-1.2	-48.5	-2.8	2.3	13.8
FourM	-10.1	-6.2	-2.0	-1.2	-7.1	-5.1	1.4	8.7
Quart	-16.4	-4.1	-1.9	-0.8	-14.0	-3.0	0.4	6.5
BiM	-7.5	-3.3	-0.7	-0.9	-5.8	-2.4	1.2	3.8
Monthly	-0.9	-0.5	-0.8	-1.9	0.0	0.0	0.0	0.0

Negative (positive) entries represent a percentage decrease (increase) in RMSE compared to base (initial) forecasts.

Simulation setup 1: results

Scenario 4 (partial misspecification):
Base forecasts from automatically selected ETS models.

Sample	4	12	20	40	4	12	20	40
(h)	*(1)*	*(3)*	*(5)*	*(10)*	*(1)*	*(3)*	*(5)*	*(10)*
	WLS				**Bottom-up**			
Annual	-24.7	1.6	0.5	-1.8	-20.9	69.1	101.5	150.4
SemiA	-42.5	-5.4	-2.7	-1.1	-40.0	35.4	63.8	105.3
FourM	-9.4	-6.7	-2.7	-4.3	-5.7	23.4	47.8	73.1
Quart	-1.2	-8.3	-5.5	-5.9	2.3	15.5	33.3	54.9
BiM	-0.9	-8.3	-9.3	-8.6	1.9	8.2	16.1	32.7
Monthly	-1.4	-7.3	-11.3	-16.9	0.0	0.0	0.0	0.0

Negative (positive) entries represent a percentage decrease (increase) in RMSE compared to base (initial) forecasts.

Simulation setup 2

- Take one draw from DGP 1 at the monthly level and fit an ETS model: ETS(A, A_d, A).

$$
\begin{aligned}
\mu_t &= \ell_{t-1} + \phi b_{t-1} + s_{t-m} \\
\ell_t &= \ell_{t-1} + \phi b_{t-1} + \alpha\varepsilon_t \\
b_t &= \phi b_{t-1} + \beta\varepsilon_t \\
s_t &= s_{t-m} + \gamma\varepsilon_t \\
\hat{Y}_{t+h|t} &= \ell_t + \phi_h b_t + s_{t-m+h_m^+}
\end{aligned}
$$

where $\phi_h = \phi + \cdots + \phi^h$ and $h_m^+ = \big[(h-1) \bmod m\big] + 1$

$\hat{\alpha} = \hat{\beta} = 0.0144, \hat{\gamma} = 0.5521, \hat{\phi} = 0.9142.$

1. Scenario 1: Forecast with ETS;
2. Scenario 2: Forecast with ARIMA;

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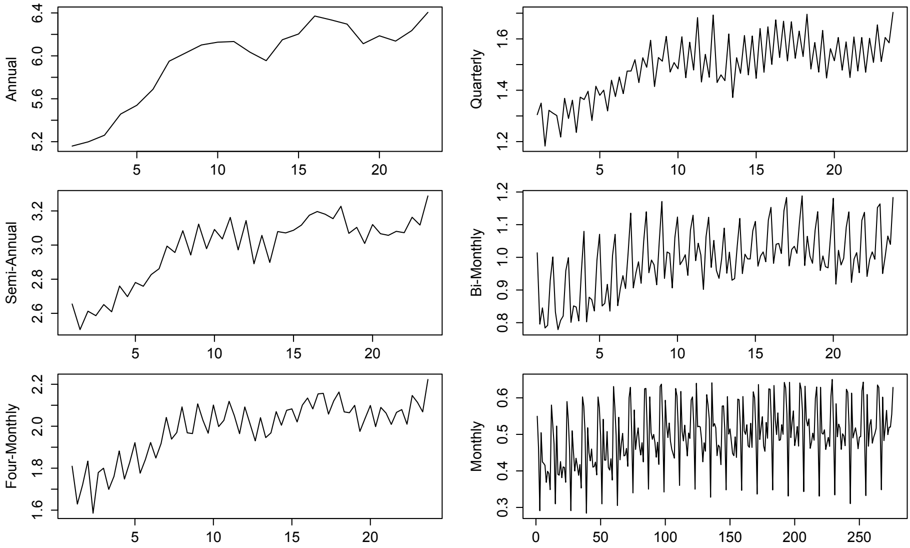
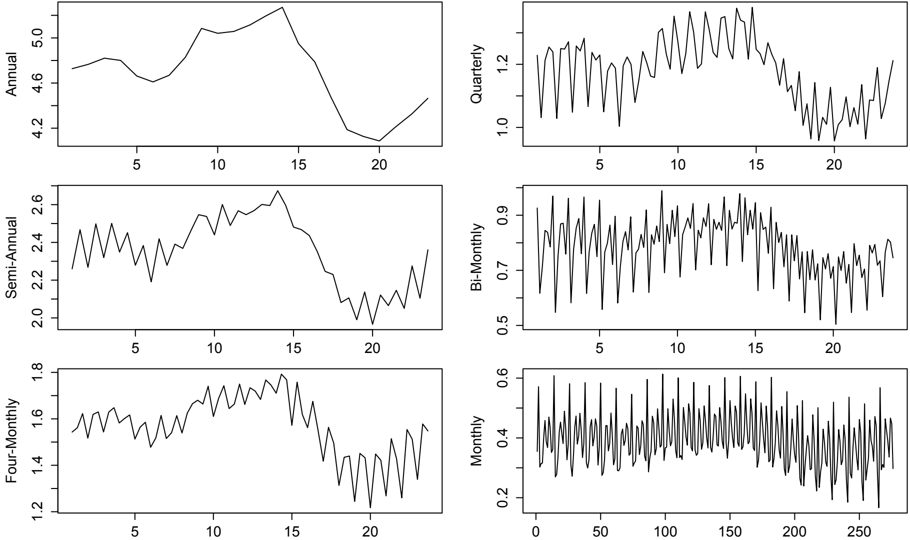
ETS(A, A_d, A)


Annual
6.4
6.0
5.6
5.2
5
10
15
20
Quarterly
1.6
1.4
1.2
Semi-Annual
3.2
3.0
2.8
2.6
Bi-Monthly
1.2
1.1
1.0
0.9
0.8
Four-Monthly
2.2
2.0
1.8
1.6
Monthly
0.6
0.5
0.4
0.3
0
50
100
150
200
250

ETS(A, A_d, A)



Annual
Quarterly
Semi-Annual
Bi-Monthly
Four-Monthly
Monthly

Simulation setup 2: results

Scenario 1: DGP is ETS(A, A_d, A). Fitting ETS.

Sample	4	12	20	40	4	12	20	40
(h)	*(1)*	*(3)*	*(5)*	*(10)*	*(1)*	*(3)*	*(5)*	*(10)*
	WLS				**Bottom-up**			
Annual	-12.3	-5.3	-7.1	-9.8	-7.0	1.2	-6.7	-6.4
SemiA	-26.9	-3.5	-5.6	-4.2	-23.5	4.2	-5.2	-0.9
FourM	-5.2	-3.6	-5.4	-1.5	-1.4	4.3	-5.0	1.6
Quart	-2.3	-4.5	-5.0	-0.9	1.1	3.2	-4.8	2.1
BiM	-1.4	-4.0	-1.9	0.3	1.1	3.3	-1.8	3.0
Monthly	-1.4	-4.7	-0.1	-1.9	0	0	0	0

Negative (positive) entries represent a percentage decrease (increase) in RMSE compared to base (initial) forecasts.

Simulation setup 2: results

Scenario 2: DGP is ETS(A, A_d, A). Fitting ARIMA.

Sample	4	12	20	40	4	12	20	40
(h)	*(1)*	*(3)*	*(5)*	*(10)*	*(1)*	*(3)*	*(5)*	*(10)*
	WLS				**Bottom-up**			
Annual	-39.9	-7.6	-9.4	-1.0	-36.4	-2.0	-2.6	5.8
SemiA	-36.6	-1.3	-2.1	-0.8	-33.6	3.7	4.4	6.1
FourM	-12.6	-4.0	-3.8	-2.6	-8.8	0.7	2.2	3.9
Quart	-23.9	-3.9	-4.4	-5.1	-19.8	0.3	1.2	1.3
BiM	-11.5	-2.9	-3.6	-3.6	-8.2	0.5	1.7	2.6
Monthly	-2.9	-2.5	-3.8	-5.0	0	0	0	0

Negative (positive) entries represent a percentage decrease (increase) in RMSE compared to base (initial) forecasts.

Outline

1 Introduction

2 Temporal hierarchies

3 Optimal combination forecasts

4 A Monte-Carlo simulation study

5 Conclusion

Conclusion/Implications

1. Significant forecast gains from applying temporal hierarchies both in simulations and empirical evaluations.
2. Beside the forecast gains we achieve the alignment of short, medium and long term forecasts.
3. Significant implications from an operational, managerial point of view.

Interesting questions

- Measurement error versus the level of aggregation.
- Aggregation level versus the effect of outliers.
- Taking advantage of high frequency being updated more regularly.

Thank you!