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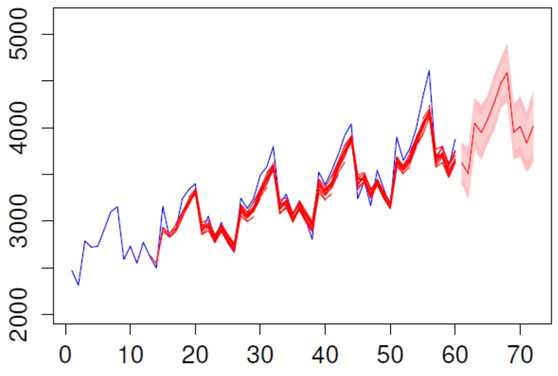
Forecasting with R
A practical workshop



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Forecasting with R

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Contents

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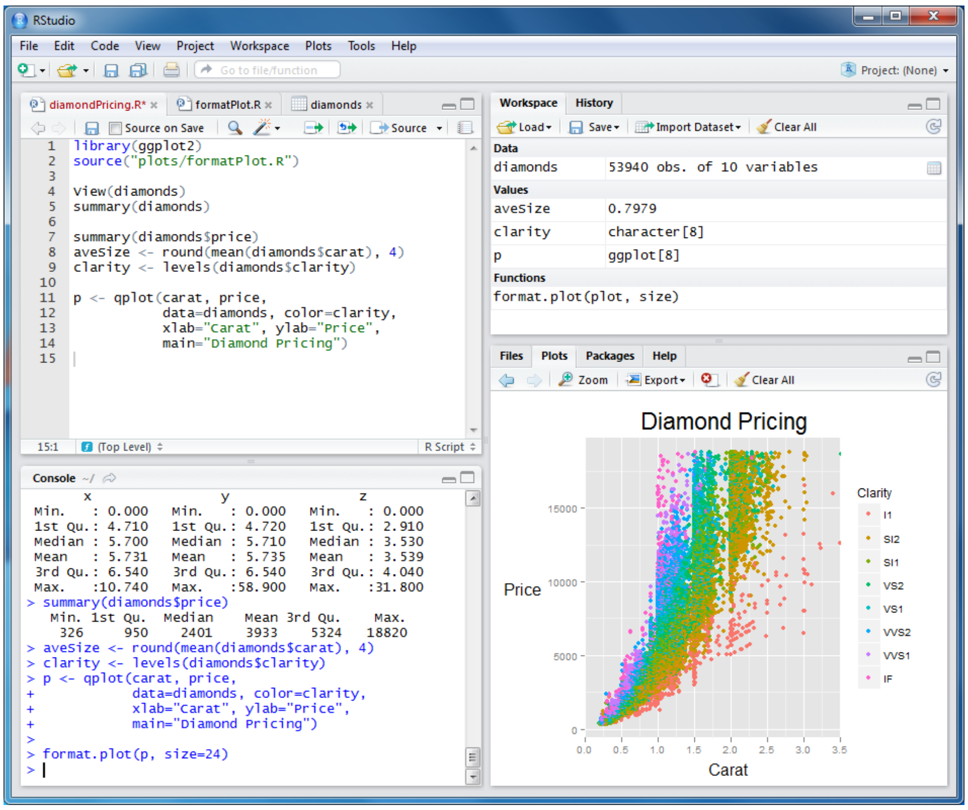
1. Overview of RStudio

RStudio is a freeware integrated development environment (IDE) for the R statistical language. It provides a simple and intuitive user interface for scripting, loading and saving results and producing graphs. Figure 1.1 provides a screen-shot of the RStudio. This consists of the following four panels:



```
library(ggplot2)
source("plots/formatPlot.R")

View(diamonds)
summary(diamonds)

summary(diamonds$price)
aveSize <- round(mean(diamonds$carat), 4)
clarity <- levels(diamonds$clarity)

p <- qplot(carat, price,
           data=diamonds, color=clarity,
           xlab="Carat", ylab="Price",
           main="Diamond Pricing")

```

```
       x                y                z
 Min.   : 0.000   Min.   : 0.000   Min.   : 0.000
 1st Qu.: 4.710   1st Qu.: 4.720   1st Qu.: 2.910
 Median : 5.700   Median : 5.710   Median : 3.530
 Mean   : 5.731   Mean   : 5.735   Mean   : 3.539
 3rd Qu.: 6.540   3rd Qu.: 6.540   3rd Qu.: 4.040
 Max.   :10.740   Max.   :58.900   Max.   :31.800
> summary(diamonds$price)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
    326     950    2401    3933    5324   18820
> aveSize <- round(mean(diamonds$carat), 4)
> clarity <- levels(diamonds$clarity)
> p <- qplot(carat, price,
+            data=diamonds, color=clarity,
+            xlab="Carat", ylab="Price",
+            main="Diamond Pricing")
>
> format.plot(p, size=24)
> |
```

```
Data
diamonds          53940 obs. of 10 variables
Values
aveSize           0.7979
clarity           character[8]
p                 ggplot[8]
Functions
format.plot(plot, size)
```

Figure 1.1: A screen-shot of RStudio, an integrated development environment for R statistical software.

1. The **top-left** panel is a text editor where the R scripts can be compiled. *Tip: you can run the current line of the script or the current selection by pressing Ctrl+Enter.*
2. The **bottom-left** panel is the console where all messages and outputs are printed.
3. The **top-right** panel consists of the tabs environment (where all variables and objects are saved and presented) and history (past commands previously ran).
4. The **bottom-right** panel consists of the tabs files (which provides access to the file system), plots (where the produced graphs are displayed), packages (where packages can be installed/uninstalled, updated and loaded), and help (which provides documentation for the packages and functions).

An important folder when working with R and RStudio is the working directory, which is a physical computer folder from which data files can be read and/or saved. To set the working directory within RStudio, from the main menu go to Session/Set Working Directory/Choose Directory, or simply type:

```r
setwd("C:/MyFolder/")
```

where “MyFolder” should be replaced with the actual file-path.

2. Introduction to R

R is a powerful statistical tool that can be used for any kind of statistical analysis (and not only forecasting). This section will provide an overview to R statistical language and some simple functions, programming tools, including the *if-statement* and *for-loop*, and data structures.

2.1. R as calculator

First, let's try to use R as a calculator:

```
(10+8)/4 #simple calculations using R
4*5^2 #multiplication and power
7 / 2 #division
7 %/% 2 #integer division
7 %% 2 #remainder of integer division
1/0
0/0
sqrt(4) #function of square root
x <- sqrt(4 * max(-3, 9, 0.8)) #saving a result into a variable
flag <- TRUE #boolean variable
```

The output in the console is:

```
[1] 4.5
[1] 100
[1] 3.5
[1] 3
[1] 1
[1] Inf
[1] NaN
[1] 2
```

while the value of `x` in the environment (top-right panel) is 6 and the value of `flag` is `TRUE`.

2.2. R as programming language

R, as a programming language, allows for several programming statements. The two most useful ones are the *if-statement* and the *for-loop*. The former allows for conditional running of some commands, while the latter is suitable for repeated calculations (such as the calculation of the expression $\sum_{i=1}^{20} i^3$).

```
b <- 5
c <- 3

if (b < 4){
  d <- b+c #this will run only if the condition (b<4) is true
}

if (b <= 4){
  d <- b+c #this will run if the condition (b<=4) is true
} else {
  d <- b-c #this will run if the condition is false
}

for (i in 1:10){
  print("Hello!") #prints the message "Hello!" ten times
}

s <- 0 #initialise sum with zero
for (i in 1:20){
  s <- s + i^3 #calculate the expression
}
```

In the first *if-statement* the condition is `FALSE`, so the variable `d` does not take any values. In the second *if-statement* the condition is `FALSE` again, so the else-command is run and the value of `d` is 2. The value of variable s after the end of the second *for-loop* is 44100.

Boolean operators are useful in constructing conditions: $<$, $>$, $==$ $(=)$, $!=$ $(\neq)$, $<=$ $(\leq)$ and $>=$ $(\geq)$. Also, two or more conditions can be combined together with & (and) or | (or).

2.3. *Data structures*

R can handle a large number of data structures. The simplest one are single-dimensional arrays, or vectors. Data values in vectors can be specified manually or read from external files. *Tip: make sure you have set as the working directory the folder that contains the data.*

```
y = c(4,6,2,5,3,7,10) #a vector containing 7 items (elements)
z = scan("z_vector.txt") #another vector of size 7 that is read from an
   external file
y[2] #returns the 2nd element of y
y[4:6] #returns elements 4, 5 and 6 of y
z[7] #returns the 3rd element of z
y+z #adds each element of y with the respective element of z
y*z #multiplies each element of y with the respective element of z
y+3 #adds 3 to each element of y
y/2 #divides each element of y by 2
```

The first two commands save (in the environment) two vectors each of size 7. The output for the rest of the lines is presented below:

```
[1] 6
[1] 5 3 7
[1] 5
[1] 10 8 8 9 6 8 15
[1] 24 12 12 20 9 7 50
[1] 7 9 5 8 6 10 13
[1] 2.0 3.0 1.0 2.5 1.5 3.5 5.0
```

More complicated data structures include the matrices, arrays (of any dimension) and data frames.

```
# Read data from an external file and arrange as a matrix of order 3x5
#"byrow" argument
m <- matrix(scan("m_matrix.txt"), nrow=3, ncol=5, byrow=TRUE)
m #diplays the matrix

# Define a 3-d array populated with random numbers (sample() function)
b <- array(sample(30), c(3,5,2))
b #displays the 3-d array

# Create two vectors, one with strings and one with integers
name <- c("John", "Jennifer", "Andrew", "Peter", "Christine")
age <- c(30, 45, 23, 56, 32)
# Combine the two vectors in a data frame
data.frame(name, age)
```

R's output is:

```
     [,1] [,2] [,3] [,4] [,5]
[1,]   4    5    6    3    1
[2,]   6   10   11    5    2
[3,]   9    7   -4    0    1

#array b is different for each run, given that it is randomly populated

       name  age
1      John   30
2  Jennifer   45
3    Andrew   23
4     Peter   56
5 Christine   32
```

2.4. Reading data

We will now load some data and transform them into time series format. Then, we will plot these data.

```
# Load quarterly data and convert to time series
ts1 <- ts(scan("ts1.txt"), start=c(2011,1), frequency=4)
# Plot the data, adding a main title and a suitable label for the y-axis
plot(ts1, main="Stationary quarterly data", ylab="Demand")

# Load monthly data and convert to time series
ts2 <- ts(scan("ts2.txt"), start=c(2011,1), frequency=12)
plot(ts2, main="Trend and seasonal monthly data", ylab="Demand")

ts3 <- ts(scan("ts3.txt"), start=c(2011,1), frequency=12)
plot(ts3, main="Intermittent monthly data", ylab="Demand")
```

The produced plots are presented in figure 2.1.

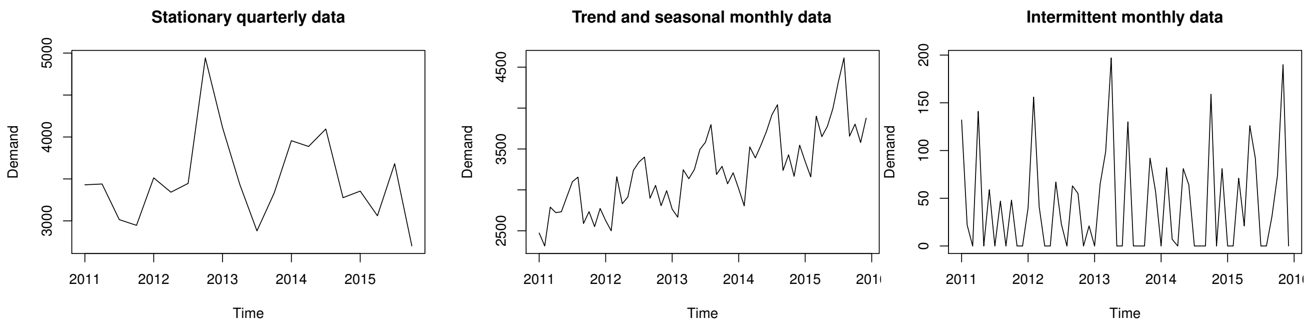


Figure 2.1: Visualisation of the three time series.

2.5. Useful functions

Some useful and generic functions in R include the sequence (`seq`), repeat (`rep`), minimum (`min`), maximum (`max`), summation (`sum`), arithmetic mean (`mean`), median (`median`), mean absolute deviation (`mad`), variance (`var`), and standard deviation (`sd`).

```
seq(1, 3, 0.25) #sequence function
rep(4, 10) #repeat function

min(y) #minimum value
max(y) #maximum value

# Measures of central tendency
mean(y) #arithmetic mean
median(y) #median
table(y) #for finding the mode

# Measures of dispersion
mad(y) #mean absolute deviation
var(y) #variance
sd(y) #standard deviation
```

The output in the console is:

```
[1] 1.00 1.25 1.50 1.75 2.00 2.25 2.50 2.75 3.00
[1] 4 4 4 4 4 4 4 4 4 4
[1] 2
[1] 10
[1] 5.285714
[1] 5
y
 2  3  4  5  6  7 10
 1  1  1  1  1  1  1
[1] 2.9652
[1] 7.238095
[1] 2.690371
```

3. Time series exploration

In this section we will look how to perform basic time series exploration using R.

3.1. Packages and functions

We will be using the **forecast** and **TStools** packages. We can load these by typing:

```
#load the necessary libraries
library(forecast)
library(TStools)
```

Table 3.1 lists the functions[1] that we will be using:

Table 3.1: Functions used for time series exploration

Function	Package	Description
cmav	Tstools	Calculate Centred Moving Average
coxstuart	TStools	Cox-Stuart test
seasplot	TStools	Visualise and test seasonality
seasonplot	forecast	Visualise seasonality
decomp	TStools	Classical decomposition
residout	TStools	Control chart for residuals
stl	forecast	STL decomposition
acf	stats	Autocorrelation function
pacf	stats	Partial autocorrelation function
diff	stats	Time series differencing

3.2. Time series components

In the first part of our exploration we will look for the presence of trend and seasonality in a time series. We will do this both visually and by using statistical tests. First let us load some data and plot the time series:

```
ts2 <- ts(scan("ts2.txt"), start=c(2011,1), frequency=12)
# Let us store the time series to be explored in variable 'y' so that we can
# repeat the analysis easily with new data if needed.
y <- ts2

# First we plot the series to get a general impression
plot(y)
```

[1]If you have problems using packages hosted in Github, like TStools, keep in mind that you can download individual functions and load them using: `source("function name")`. For example, to load cmav one can write `source("cmav.R")`.

To calculate the Centred Moving Average (CMA) of a time series we will use the function `cmav`. This can accept time series (`ts`) objects or vectors. In the first case the CMA length is automatically set to be equal to the sampling frequency. In the second case, or if we want to override this setting, we can use the option `ma=X`, where `X` is the desired length.

```
# Let us look for trend in the data, by calculating the Centred Moving
# Average
cma <- cmav(y, outplot=1)
print(cma)
```

The console output is:

```
> cma <- cmav(y, outplot=1)
> print(cma)
          Jan      Feb      Mar      Apr      May      Jun      Jul      Aug      Sep      Oct
              Nov      Dec
2011 2659.533 2673.576 2687.619 2701.662 2715.705 2729.748 2743.792 2757.833
    2781.042 2801.042 2813.125 2834.042
2012 2857.417 2877.625 2900.708 2926.958 2951.000 2970.708 2985.583 2998.333
    3008.833 3025.167 3051.875 3076.500
...
```

The CMA is plotted as a red line, as in figure 3.1, where we can clearly see that there is an upwards trend in the time series. The function `cmav` by default backcasts and forecasts

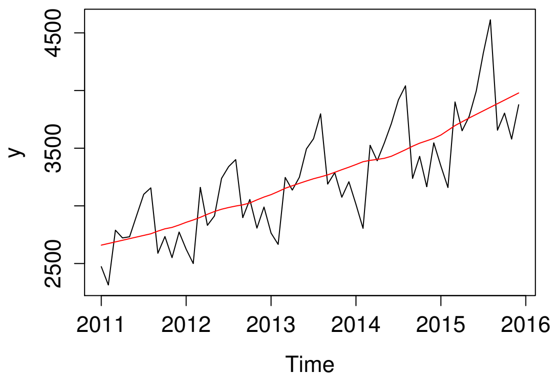


Figure 3.1: Plot of CMA.

the missing starting and ending values using exponential smoothing (`ets` from the *forecast package*). If we want to stop this we can use the following:

```
# CMA without back- and forecasting of missing values
cmav(y, outplot=1, fill=FALSE)
```

This will give use a result as in figure 3.2. We can test whether the estimated trend is significant. There are many tests to do this, but we prefer the non-parametric (but relatively

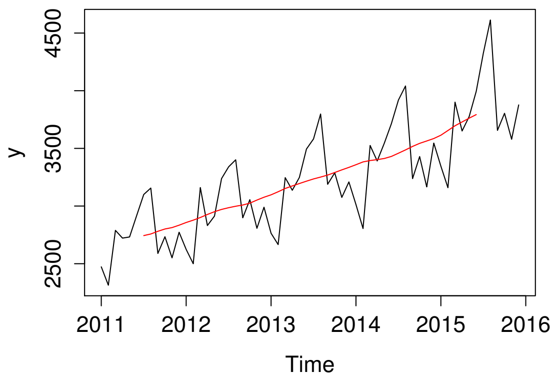


Figure 3.2: Plot of CMA without backcasting and forecasting the missing values.

weak) Cox-Stuart test.

```
# Cox-Stuart on the estimated CMA to test for significant trend
coxstuart(cma)
```

The console output of the test is:

```
> coxstuart(cma)
$H
[1] 1

$p.value
[1] 9.313226e-10

$Htxt
[1] "H1: There is trend (upwards or downwards)"
```

Next we will explore for seasonality. When trend is present it is good practice to first remove it. The function that we will be using to visualise and test for seasonality does this automatically, but the user can also force a specific behaviour if needed.

```
# We can test for seasonality visually by producing a seasonal plot
seasplot(y)
# This functions removes the trend automatically but we can control this.
# It also provides other interesting visualisations of the seasonal component
seasplot(y,outplot=2)
seasplot(y,outplot=3)
seasplot(y,outplot=4)
```

The various visualisations are given in figure 3.3. The seasonal boxplot is useful to see how we could test for the presence of seasonality. If the location of each monthly distribution is significantly different form the rest that would indicate presence of seasonality. This function

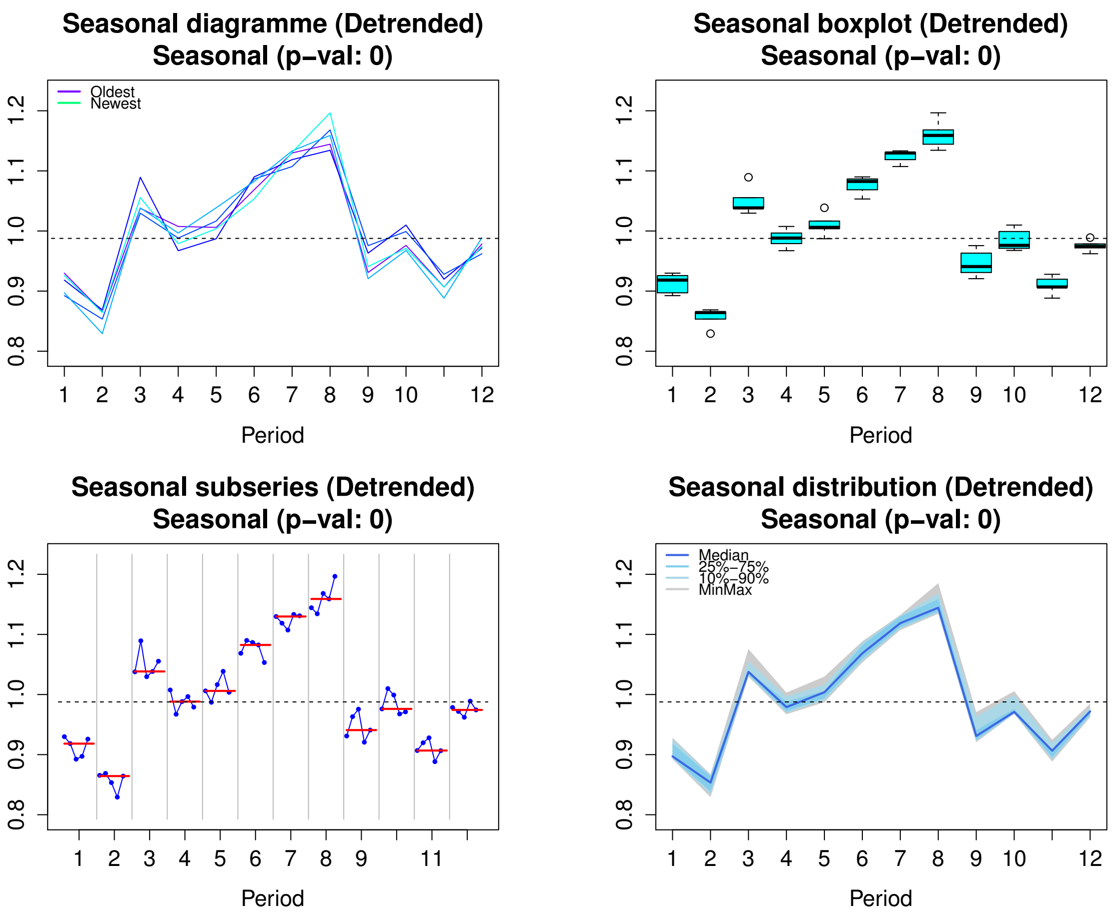


Figure 3.3: Variations of the `seasonplot` output plot.

tests for that using the non-parametric Friedman test, the p-value of which is reported in the plots (see figure 3.3) and in the console output, which provides the estimated seasonal indices, trend (if any) and the respective p-values for testing for presence of these.

```
$season
          1         2         3         4         5        6         7        8         9        10
                 11        12
s1 0.9298624 0.8655074 1.037721 1.0075279 1.006000 1.068597 1.129823 1.144377
   0.9309461 0.9760655 0.9068207 0.9784613
s2 0.9183120 0.8687720 1.089389 0.9672157 0.987123 1.089976 1.118709 1.134297
   0.9631640 1.0098617 0.9197624 0.9718836
...
s5 0.9258012 0.8643152 1.055418 0.9790282 1.003710 1.053244 1.131002 1.196615
   0.9408029 0.9711409 0.9065252 0.9743549
s6        NA        NA       NA        NA       NA       NA       NA       NA        NA        NA
          NA        NA

$season.exist
[1] TRUE

$season.pval
[1] 1.749586e-07

$trend
         Jan      Feb      Mar      Apr      May      Jun      Jul      Aug      Sep      Oct
             Nov      Dec
2011 2659.533 2673.576 2687.619 2701.662 2715.705 2729.748 2743.792 2757.833
   2781.042 2801.042 2813.125 2834.042
2012 2857.417 2877.625 2900.708 2926.958 2951.000 2970.708 2985.583 2998.333
   3008.833 3025.167 3051.875 3076.500
...

$trend.exist
[1] TRUE

$trend.pval
[1] 9.313226e-10
```

An alternative visualisation is offered by the *forecast* package, the output of which can be seen in figure 3.4

```
# The equivalent function in the forecast package is:
seasonplot(y)
```

The main differences between the functions are in the additional options that `seasplot` offers on removing the trend, estimation of seasonal indices and visualisations.

3.3. Time series decomposition

We can perform classical decomposition by using the function `decomp`. This allows us to control the trend and the estimation of the seasonal component. For example we

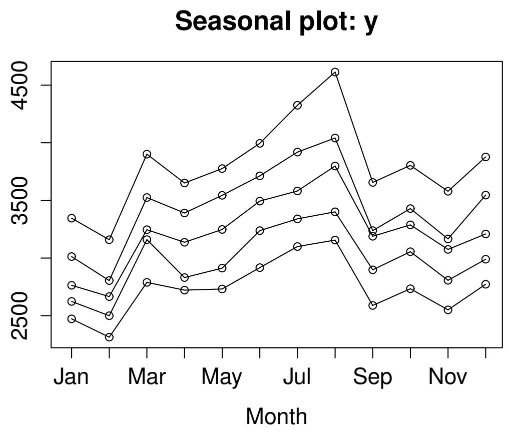


Figure 3.4: `seasonplot` output. Observe that the time series trend is not removed.

can use seasonal smoothing instead of the arithmetic mean (or median) to estimate the seasonal indices and ask it to predict future seasonal values as well. This can be quite useful for decomposing the time series to forecasting with methods such as Theta and then superimposing the seasonal component.

```
# We can perform classical decomposition using the decomp function
decomp(y,outplot=1)
# or using the pure.seasonal for estimating the seasonal indices
y.dc <- decomp(y,outplot=1,type="pure.seasonal",h=12)
```

If we use the option `outplot=1` then we will get a visualisation as in figure 3.5, where the forecasted seasonal indices are also plotted. The function outputs the numerical values of the trend, seasonal and irregular components. If seasonal smoothing is used then the parameters of the fitted model are also provided.

We can visualise the residuals for any extraordinary behaviour:

```
# Control chart of residuals
residout(y.dc$irregular)
```

which provides the plot in figure 3.6.

The console output highlights any extraordinary (by default over 2 standard deviations away from zero) values:

```
$location
[1] 15 56

$outliers
[1] 2.170237 2.637928

$residuals
          Jan       Feb       Mar       Apr       May       Jun       Jul       Aug
                    Sep       Oct       Nov       Dec
```

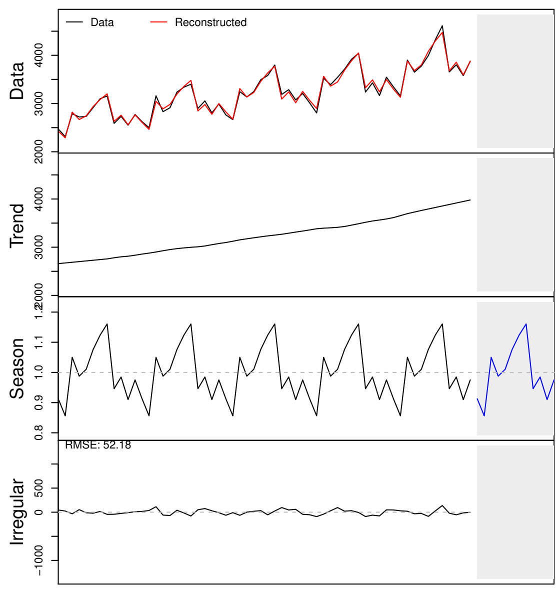


Figure 3.5: Classical decomposition using seasonal smoothing to estimate the seasonal indices and providing forecasts for the next 12 periods.

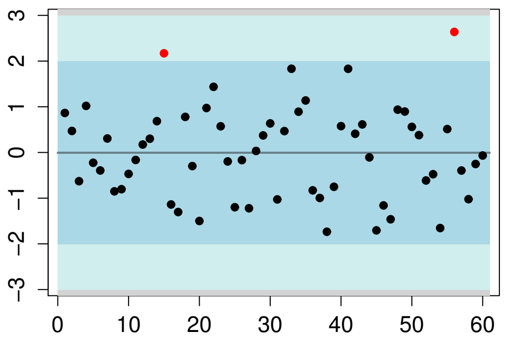


Figure 3.6: Control chart of residuals. Different shades define different ranges of standard deviation.

```
2011 0.86412086 0.47036905 -0.62841740 1.01930127 -0.22539447 -0.39399424
    0.30564341 -0.85094357 -0.80378649 -0.46907466 -0.16421443 0.17351057
2012 0.30113730 0.68481526 2.17023690 -1.13825701 -1.30368743 0.77830399
    -0.29807646 -1.49956077 0.97278234 1.43655102 0.57251943 -0.19625191
...
```

This threshold can be controlled using the option `t=X`, where `X` is the number of standard deviations desired.

The *forecast* package offers an alternative that is based on loess estimation of the trend:

```
# STL time series decomposition
y.stl <- stl(y,s.window=7)
plot(y.stl)
```

The output of which is plotted in figure 3.7. Note that one has now to set the parameters for the loess fit, which has no defaults.

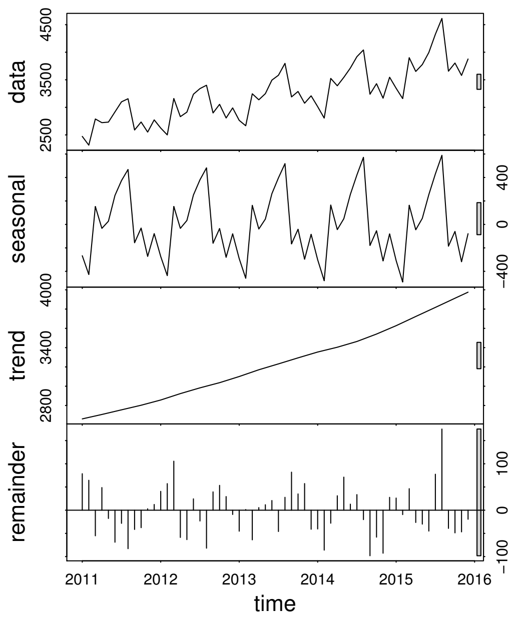


Figure 3.7: The output of STL decomposition.

Finally, The *seasonal* package offers an interface for X-13-ARIMA-SEATS for a more advanced time series decomposition.

3.4. Autocorrelation and partial autocorrelation functions

The build-in functions `acf` and `pacf` are useful to calculate the autocorrelation and partial autocorrelation functions respectively. Figure 3.8 presents the output of these functions. We can control how many lags to visualise using the option `lag.max`.

```
# Calculate and plot ACF and PACF
acf(y,lag.max=36)
pacf(y,lag.max=36)
```

We can introduce differencing as appropriate using the built-in function `diff`. For example:

```
# 1st difference
plot(diff(y,1))
acf(diff(y,1),lag.max=36)
# Seasonal difference
```

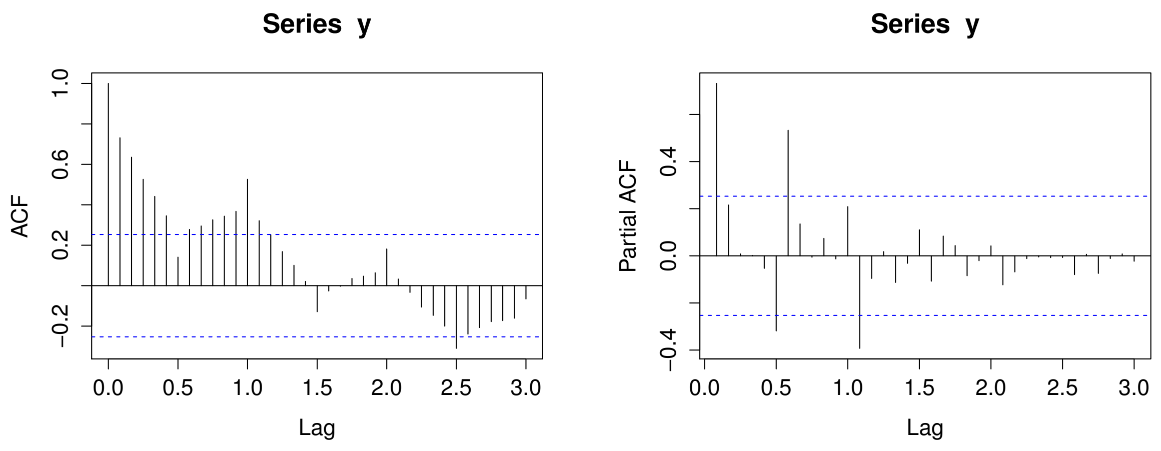


Figure 3.8: ACF and PACF plots.

```
plot(diff(y,12))
acf(diff(y,12),lag.max=36)
# 1st & seasonal differences
plot(diff(diff(y,12),1))
acf(diff(diff(y,12),1),lag.max=36)
```

---

4. Forecasting for fast demand

In this section we look how to produce univariate forecasts for conventional time series, such as fast demand data.

4.1. Packages and functions

We will be using the following three packages **forecast**, **MAPA** and **TStools** packages. Table 4.1 lists the functions that we will be using:

Table 4.1: Functions used for univariate forecasting

Function	Package	Description
ets	forecast	State-space exponential smoothing models
auto.arima	forecast	SARIMA with automatic specification
tbats	forecast	TBATS model
forecast	forecast	Predict for forecast package methods
tsaggr	MAPA	Non-overlapping temporal aggregation
mapaest	MAPA	Estimate MAPA
mapafor	MAPA	Forecast with MAPA
mapa	MAPA	Wrapper containing both mapaest and mapafor
theta	TStools	Theta method with automatic seasonal decomposition
stlf	forecast	STL decomposition and forecast
decomp	TStools	Classical decomposition
nemenyi	TStools	Friedman and Nemenyi post-hoc test
ts.plot	stats	Function to plot several time series

4.2. Preparation: load packages and time series

Before producing the forecasts let us load the relevant packages and some time series.

```r
# Load the necessary libraries
library(forecast)
library(MAPA)
library(TStools)

# Load two example time series, as before
ts1 <- ts(scan("ts1.txt"), start=c(2011,1), frequency=4)
ts2 <- ts(scan("ts2.txt"), start=c(2011,1), frequency=12)
# In order to evaluate our forecasts let us load some test data as well.
ts1.test <- ts(scan("ts1out.txt"), start=c(2016,1), frequency=4)
ts2.test <- ts(scan("ts2out.txt"), start=c(2016,1), frequency=12)
```

We first focus on the second time series (ts2) that has more interesting features. We store the in-sample and test data into two new variables which will be used for the generation of all forecasts. The idea is that we can simply replace the time series in these two variables and reuse the same code. For this example, we set the forecast horizon equal to the size of the test set.

```
# Let us first model ts2
y <- ts2
y.test <- ts2.test

# Set horizon
h <- length(y.test)
```

4.3. Naïve and Seasonal Naïve

We do not use any special functions to produce Naïve forecasts (nonetheless the forecast package offers the functions `naive` and `snaive` for that purpose if you prefer). Here we will construct the forecasts manually. The result can be seen in figure 4.1.

```
# Naive
# Replicate the last value h times
f.naive <- rep(tail(y,1),h)
# If we transform this to a ts object it makes plotting the series and the
# forecast together very easy.
# Otherwise we need to be careful with the x coordinates of the plot
f.naive <- ts(f.naive,start=start(y.test),frequency=frequency(y.test))
# Now we can plot all elements
ts.plot(y,y.test,f.naive,col=c("blue","blue","red"))
```

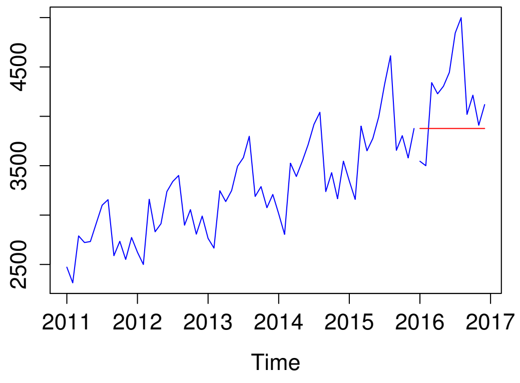


Figure 4.1: Time series and Naïve forecast.

Obviously since the time series is seasonal the result is poor. Let us instead try to produce seasonal naïve forecasts, the result of which can be seen in figure 4.2. Although this forecast captures the seasonal shape it ignores the apparent trend.

```
# Seasonal Naive
f.snaive <- rep(tail(y,frequency(y)),ceiling(frequency(y)/h))
f.snaive <- ts(f.snaive,start=start(y.test),frequency=frequency(y.test))
ts.plot(y,y.test,f.snaive,col=c("blue","blue","red"))
```

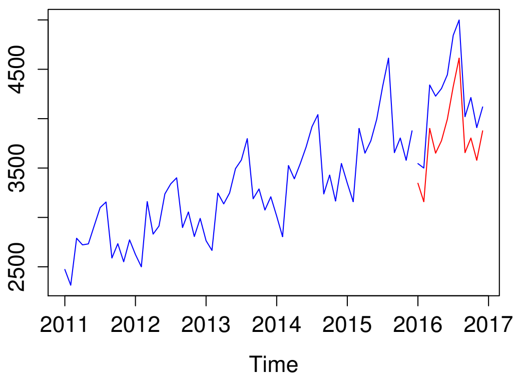


Figure 4.2: Time series and Seasonal Naïve forecast.

4.4. Exponential smoothing

Exponential smoothing is one of the most widely used extrapolative forecasting methods. The *forecast* package offers the `ets` functions that implement exponential smoothing with a state space formulation. The function is able to produce all 30 standard models, with different types of trend (none, additive, multiplicative and damped), seasonality (none, additive, multiplicative) and error term (additive and multiplicative). The models are named with the following convention ETS(Error,Trend,Season). For example ETS(A,N,N) has additive errors with no trend and seasonality, i.e. is the well known single exponential smoothing. For details about the state space implementation of exponential smoothing the reader is referred to Hyndman et al. (2008).

The `ets` function fits all possible models for the given time series, by finding optimal smoothing parameters and initial values, and selects the best using some information criterion. By default this is the Akaike's Information Criterion corrected for sample size (AICc), but other options are also available. The user can also input predefined smoothing parameters.

For our example we let the automatic procedure run:

```r
# Exponential smoothing
# First we need to fit the appropriate model, which is selected automatically
    via AICc
fit.ets <- ets(y)
print(fit.ets)

# And then we use this to forecast
f.ets <- forecast(fit.ets,h=h)
# Notice that since now we have a model we can produce analytical prediction
    intervals
print(f.ets)
# Plot the resulting forecast and prediction intervals
plot(f.ets)
# Let us add the true test values to the plot
```

```
lines(y.test,col="red")

# Note that we easily can force a specific model
ets(y,model="AAA") # Additive errors, Additive trend, Additive seasonality
```

The console output provides the fitted model and the forecast with the prediction intervals and the test set actuals, which were added to the default plot, are given in figure 4.3

```
> print(fit.ets)
ETS(M,A,M)

Call:
 ets(y = y)

  Smoothing parameters:
    alpha = 0.0488
    beta = 0.013
    gamma = 1e-04

  Initial states:
    l = 2626.29
    b = 18.8062
    s=0.9755 0.9107 0.985 0.948 1.1623 1.1222
           1.0755 1.0112 0.989 1.0524 0.8569 0.9113

  sigma: 0.0185

     AIC     AICc      BIC
767.3014 779.9525 800.8109
```

The output provides information about the selected model, parameters, initial states, as well as various fit statistics. We can see in detail what elements are contained in `fit.ets` by using the function `names` as follows:

```
> names(fit.ets)
 [1] "loglik"  "aic"       "bic"       "aicc"      "mse"       "amse"      "fit"
 [8] "residuals" "fitted"  "states"  "par"       "m"         "method"   "components"
[15] "call"    "initstate" "sigma2"  "x"

> fit.ets$method
[1] "ETS(M,A,M)"
```

Furthermore, we can produce a plot of the estimated model, which provides a decomposition of the series to the fitted components. The result of the command `plot(fit.ets)` is shown is figure 4.4.

4.5. ARIMA

Next we use ARIMA to produce forecasts using the function `auto.arima`. This function has the advantage that it can automatically specify an appropriate ARIMA model. This is

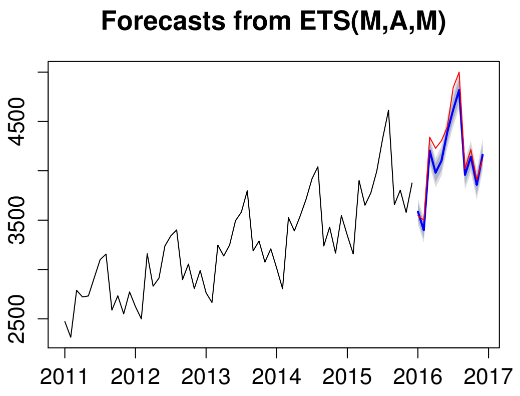


Figure 4.3: Time series and ETS forecast.

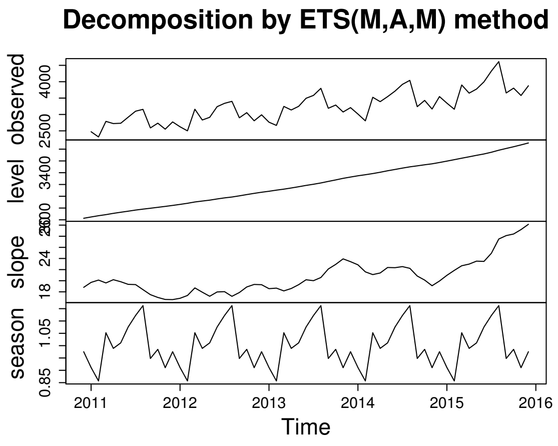


Figure 4.4: Decomposition of the time series to the fitted ETS components.

done by first using statistical tests to identify the appropriate order of differencing (including seasonal). Then once this is fixed a heuristic is used to identify the appropriate autoregressive and moving average processes orders, by tracking an appropriate information criterion. By default the AICc is used. The detailed specification methodology is described by Hyndman and Khandakar (2008). Note that to test for seasonal differencing the current version of `auto.arima` uses the OCSB test instead of the Canova-Hansen test as originally proposed. Obviously the user can pre-specify the desirable ARIMA model.

To get the ARIMA forecasts we type:

```
# ARIMA
# We can build ARIMA using the auto.arima function from the forecast package
fit.arima <- auto.arima(y)
print(fit.arima)
# We produce the forecast in a similar way
f.arima <- forecast(fit.arima,h=h)
```

```
plot(f.arima)
lines(y.test,col="red")
```

The console output reports the fitted model:

```
> fit.arima
Series: y
ARIMA(1,1,1)(0,1,0)[12]

Coefficients:
         ar1      ma1
      0.3698  -0.8957
s.e.  0.1780   0.1001

sigma^2 estimated as 8754: log likelihood=-279.46
AIC=564.92 AICc=565.48 BIC=570.47
```

4.6. Trigonometric Exponential Smoothing (TBATS)

Particularly for seasonal time series the forecast package offers the TBATS model, which was proposed by De Livera et al. (2011). TBATS uses a trigonometric parsimonious representation of seasonality, instead of conventional seasonal indices, and also incorporates ARMA errors. The function `tbats` also automatically performs Box-Cox transformation of the time series, as required.

Because of the parsimonious trigonometric representation of seasonality, TBATS has two major advantages: (i) it can easily incorporate multiple seasonal cycles; and (ii) the period of each seasonality does not have to be integer.

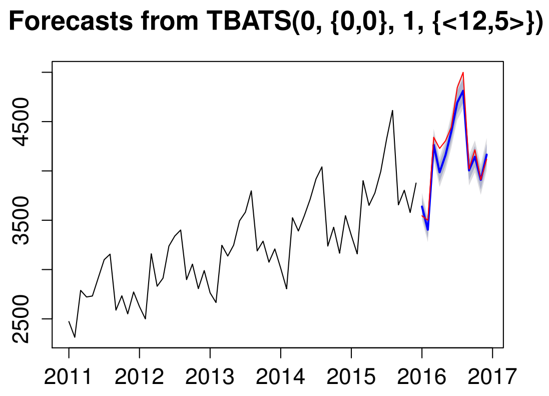
An example of using TBATS is shown below. Note that the seasonality periodicity does not need to be specified explicitly. The results of the forecast and the corresponding time series decomposition are shown in figure 4.5

```
# TBATS
fit.tbats <- tbats(y)
f.tbats <- forecast(fit.tbats,h=h)
plot(f.tbats)
lines(y.test,col="red")

# Similar to ETS, TBATS also offers a decomposition of the series
plot(fit.tbats)
```

4.7. Multiple Aggregation Prediction Algorithm (MAPA)

Recently there has been a revival of research in temporal aggregation. The *MAPA* package offers an implementation of the MAPA (Kourentzes et al., 2014), as well as supporting functions for non-overlapping temporal aggregation.

We can produce temporally aggregated versions of a time series using the `tsaggr` function. For example suppose we want to produce the temporally aggregated versions at a



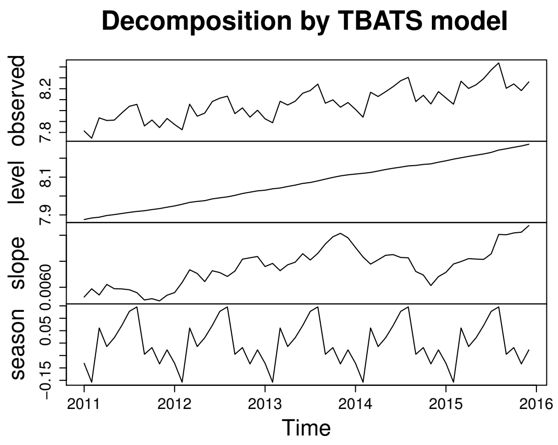


Figure 4.5: Forecast and decomposition using TBATS.

quarterly and annual level: `tsaggr(y, c(3,12))`. The function produces three sets of outputs: (i) `$out` that contains the aggregated time series; (ii) `$all` that contains an array containing all aggregated series in the original frequency; and (iii) `$idx` that contains a list of indices used to produce `$out` from `$all`.

```
> tsaggr(y, c(3,12), outplot=1)
$out
$out$AL3
  Qtr1 Qtr2 Qtr3 Qtr4
1  7576  8371  8845  8058
2  8284  8982  9639  8852
3  8676  9879 10569  9572
4  9343 10648 11198 10141
5 10406 11422 12594 11260

$out$AL12
Time Series:
Start = 1
End = 5
Frequency = 1
[1] 32850 35757 38696 41330 45682


$all
       AL3  AL12
 [1,]  7576 32850
 [2,]  7576 32850
 ...
[60,] 11260 45682

$idx
$idx$AL3
 [1]  1  4  7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58
```

```
$idx$AL12
[1] 1 13 25 37 49
```

Using the option `outplot=1` provides a visual output of the aggregated series, as seen in figure 4.6.

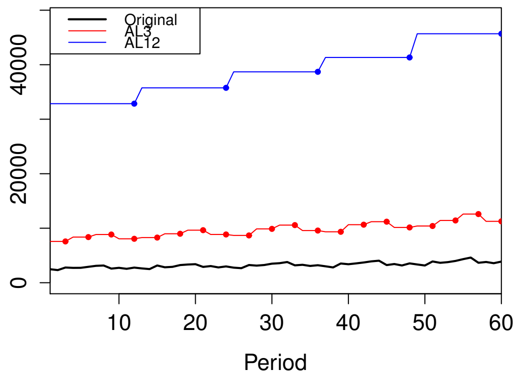


Figure 4.6: Visualisation of the resulting aggregated time series. The `$all` output is plotted.

The idea behind MAPA is to fit an appropriate exponential smoothing model at several temporally aggregated version of a time series and combine the resulting fits by components. The motivation for this is that temporal aggregation acts as a filter, strengthening or attenuating different components of the time series. Effectively it is easier to identify and fit the high-frequency components, such as seasonality, on more disaggregate views of the data, and conversly low-frequency components, like trend, are easily modelled at a more aggregate view.

```
# MAPA
# The forecasts can be produced in two steps, first mapaest is used to
# estimate the models at each aggregation level and mapafor is used to
# produce the forecasts and combine them by time series component.
fit.mapa <- mapaest(y)
f.mapa <- mapafor(y, fit.mapa, fh = h)
```

Figure 4.7 presents the various ETS models fitted at each aggregation level, as well as the resulting fit and forecast. It can be observed that at lower temporal aggregation levels the seasonal component of the time series dominates, while at higher levels it is filtered out.

Function `mapa` acts as a wrapper that does both the estimation and forecasting in a single step. Also it can be used (similarly to `mapafor`) to produce rolling in-sample forecasts and empirical prediction intervals, as shown below; the result can been seen in figure 4.8.

```
# The estimation and forecasting can be done with a single function
mapa(y, fh=h, ifh=h, conf.lvl=c(0.8,0.95))
```

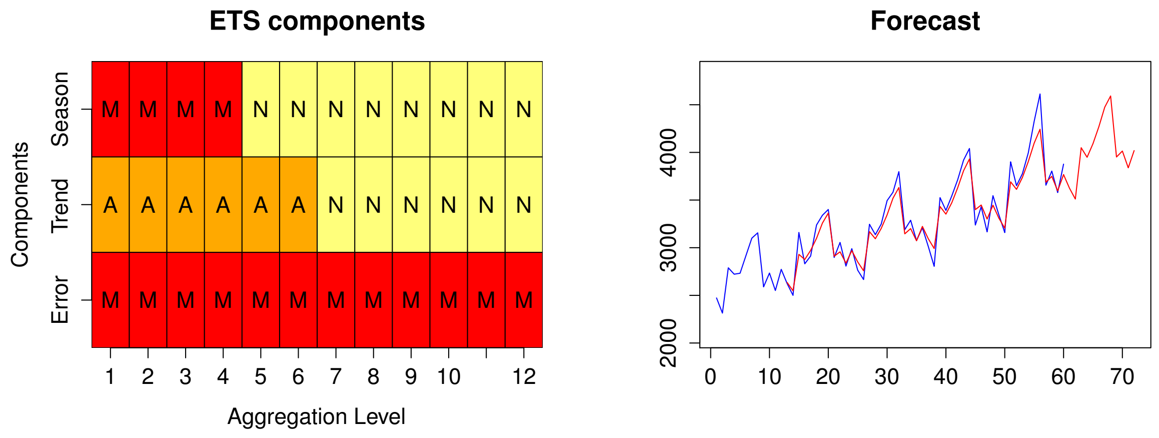


Figure 4.7: Fitted models at each aggregation level and final forecast using MAPA.

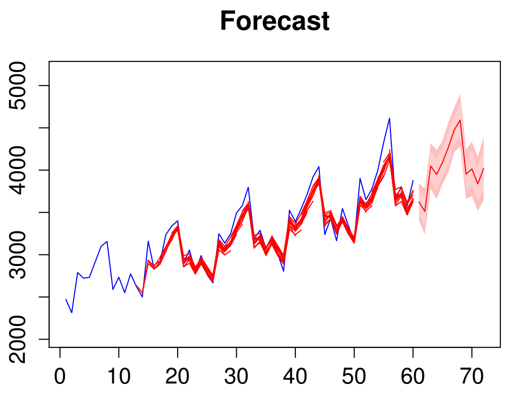


Figure 4.8: Rolling in-sample predictions and out-of-sample forecasts with empirical prediction intervals for MAPA.

4.8. Theta method

The Theta method (Assimakopoulos and Nikolopoulos, 2000) was shown to be very accurate in the M3 competition and various works since. The key aspect of the method is the decomposition of the data into two theta lines that are used to predict the seasonally adjusted time series.

The *TStools* package offers an implementation of Theta with various refinements: (i) the series is tested for presence of trend and modelled accordingly; (ii) the seasonal component is modelled using exponential smoothing; (iii) the user can control the type of decomposition used and the cost function used to estimate the method parameters.

```
# Theta
f.theta <- theta(y,outplot=1,h=h)$frc
lines(y.test,col="red")
# We can also plot the various Theta lines
```

```
theta(y,outplot=2)
```

The visual output for the code above is presented in figure 4.9.

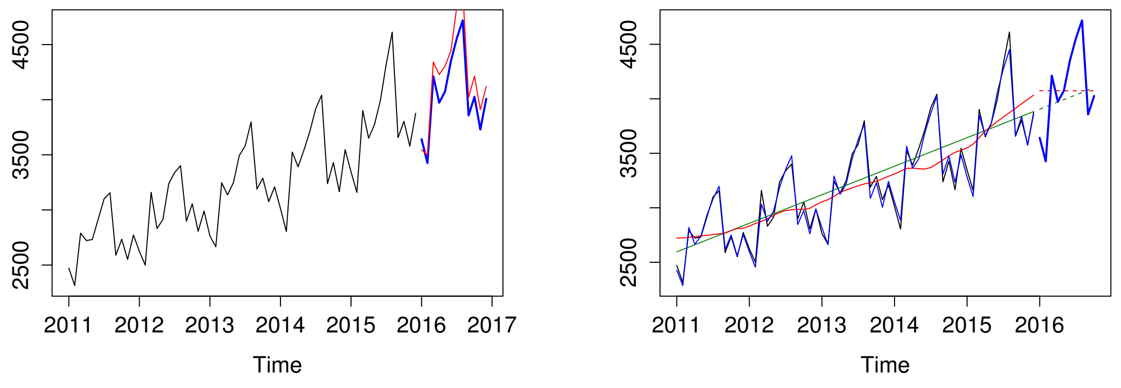


Figure 4.9: Forecast and method fit using Theta.

Fiorucci et al. (2016) showed ways to optimise the Theta method and its connection with state space models. The authors released the *forecTheta* package for R that implements the refined Theta method.

4.9. Forecasting using decomposition

Another approach that one could employ is to decompose the time series (similarly to Theta), but use any model to predict the series. This is particularly useful for high-frequency data, as many implementations do not handle long seasonalities. For example `ets` is restricted to seasonal periods up to 24.

The *forecast* package offers the `stlf` function which implements decomposition and forecasting of the non-seasonal part of the time series using either ETS or ARIMA.

```
# STLF
f.stlf <- stlf(y,method="arima",h=h)
plot(f.stlf)
lines(y.test,col="red")
```

Alternatively for a more general approach we can use the `decomp` function from the *TStools* package. Now the user can control the type of decomposition and estimation of the seasonal component, as well as use any model to predict the non-seasonal part.

```
y.comp <- decomp(y,h=h,type="pure.seasonal")
y.dc <- y/y.comp$season
# Plot the series without the seasonal component
plot(y.dc)
# Forecast it with exponential smoothing and add seasonality
f.decomp <- forecast(ets(y.dc,model="ZZN"),h=h)$mean * y.comp$f.season
ts.plot(y,y.test,f.decomp,col=c("blue","blue","red"))
```

4.10. Forecast evaluation

Various authors have provide functions for calculating forecast errors (for examples see existing functions in *TStools*). We take the stance that it is often very simple to manually calculate summary accuracy statistics and sometimes it requires more effort to bring the data in the required format for each function. The following example code calculates the bias (Mean Error) and accuracy (Mean Absolute Error and Mean Absolute Percentage Error) for all forecasts produced above.

```
# Note that we use only the $mean from the predictions that contain
   prediction intervals
f.all <- cbind(f.naive, f.snaive, f.ets$mean, f.arima$mean, f.theta,
   f.mapa$outfor, f.stlf$mean, f.decomp)
# Find the number of forecasts
k <- dim(f.all)[2]
# Replicate test set k times
# We can do this quickly by using the function tcrossprod
# We give it a vector of k ones and the test set that are matrix multiplied
   together
# This is equivalent to rep(1,k) %*% t(y.test)
y.all <- t(tcrossprod(rep(1,k),y.test))
# Calculate errors
E <- y.all - f.all
# Summarise errors
# ME is mean by columns
ME <- apply(E,2,mean)
# MAE
MAE <- apply(abs(E),2,mean)
# MAPE
MAPE <- apply(abs(E)/y.all,2,mean)*100
# Let us collate the results in a nice table
res <- rbind(ME,MAE,MAPE)
rownames(res) <- c("ME","MAE","MAPE%")
colnames(res) <-
   c("Naive","S.Naive","ETS","ARIMA","Theta","MAPA","STLF","Decomp")
print(round(res,2))
```

The resulting console output indicates that for this time series the best performing forecast is produced by ARIMA:

```
       Naive S.Naive    ETS  ARIMA  Theta   MAPA   STLF Decomp
ME    328.83 399.00 103.63  66.27 157.64 173.83 149.00 118.49
MAE   447.00 399.00 117.71 103.90 173.76 189.07 165.74 132.32
MAPE% 10.21   9.43   2.73   2.43   4.05   4.28   3.78   3.08
```

It is trivial to repeat the analysis for the first time series by simply updating the values of `y` and `y.test`.

The evaluation performed here is fixed origin and therefore quite unreliable as it is based on a single set of forecasts from each method. We can easily extend this to a rolling origin evaluation by using a for-loop to produce forecasts from different in-sample sets, effectively by only updating y and y.test.

Finally we can test the forecast errors for significant differences using the nemenyi function. For this example we test the absolute errors across different forecast horizons. This comparison is rather weak and it is recommended to compare multiple forecast errors across forecast origins.

```
# Nemenyi test
nemenyi(abs(E))                    # Default visualisation
nemenyi(abs(E),plottype="vmcb")   # Alternative visualisation
```

The resulting outputs are provided in figure 4.10. In the first visualisation for any forecasts joined by a vertical line there is no evidence of significant difference. In the figure, we can see three groupings of the methods, depending from where we start looking for the differences. For the second visualisation, for every forecast we can imagine two vertical lines from the edges of the plotted intervals (in the figure this is done for the best ranking forecast). For any forecast that its mean rank (plotted with •) is outside these bounds there is evidence of significant differences.

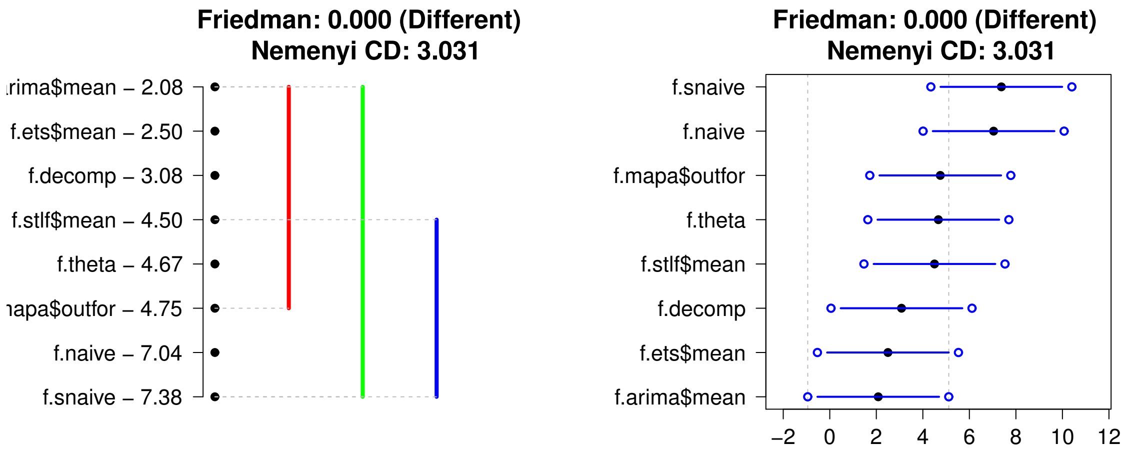


Figure 4.10: Alternative visualisations of the post-hoc Nemenyi test.

5. Forecasting for intermittent demand

In this section we will deal with forecasting slow moving demand series. This problem is quite distinct from conventional univariate forecasting and as such requires specialised functions.

5.1. Packages and functions

We will be using the *tsintermittent* package. We can load this, as well as some example time series, by typing:

```
# Load the necessary library
library(tsintermittent)

# Load the third time series
y <- ts(scan("ts3.txt"), start=c(2011,1), frequency=12)
y.test <- ts(scan("ts3out.txt"), start=c(2016,1), frequency=12)
# Set the forecats horizon to be equal to the test set
h <- length(y.test)
```

The *tsintermittent* package offers various forecasting methods for intermittent demand. Most methods are automatically optimised using the cost functions proposed by Kourentzes (2014). All functions allow the user to instead use predefined parameters and initialisation values if desired. Table 5.1 lists the functions that we will be using in this section:

Table 5.1: Functions used for intermittent series forecasting

Function	Package	Description
crost	tsintermittent	Croston's method and variants
crost.decomp	tsintermittent	Croston's method decomposition
crost.ma	tsintermittent	Croston's method using moving averages
tsb	tsintermittent	Teunter-Syntetos-Babai method
sexsm	tsintermittent	Single exponential smoothing
imapa	tsintermittent	MAPA for intermittent demand
idclass	tsintermittent	Classification of intermittent time series
simID	tsintermittent	Intermittent series generator

5.2. Croston's method and variants

Croston's method is the most widely used forecasting method for intermittent demand time series. This can be easily produced using:

```
# Croston's method
f.crost <- crost(y,h=h,outplot=1)
# The output contains various results which are documented in the function
   help.
print(f.crost)
# $frc.out is the out-of-sample forecast so we will retain only this
f.crost <- f.crost$frc.out
```

The console output for `print(f.crost)` provides the in-sample fit (`$frc.in`), the out-of-sample forecast (`$frc.out`), the parameter values (`$weights` and (`$initial`) and the non-zero demand and inter-demand interval components for the in- and out-of-sample periods (`$components`). The output of the code above is:

```
> print(f.crost)
$model
[1] "croston"

$frc.in
 [1]       NA 42.03016 41.83333 41.83333 42.46275 42.46275 42.18760 42.18760
     41.78218 41.78218 41.39217 41.39217 41.39217 40.44976 41.78823
...
[46] 41.81276 41.68598 41.68598 41.65861 41.65861 41.65861 40.93039 40.67289
    41.87612 42.60286 42.60286 42.60286 41.23357 41.71786 43.90001

$frc.out
 [1] 43.90001 43.90001 43.90001 43.90001 43.90001 43.90001 43.90001 43.90001
     43.90001 43.90001 43.90001 43.90001

$weights
[1] 0.03292563 0.03351995

$initial
[1] 132.393905 3.149974

$components
$components$c.in
        Demand Interval
 [1,]       NA       NA
 [2,] 132.39391 3.149974
 [3,] 128.75912 3.077907
 [4,] 128.75912 3.077907
...
[60,] 98.86703 2.252096

$components$c.out
       Demand Interval
 [1,] 98.86703 2.252096
 [2,] 98.86703 2.252096
 [3,] 98.86703 2.252096
 [4,] 98.86703 2.252096
 ...
[12,] 98.86703 2.252096

$components$coeff
[1] 1
```

Figure 5.1 visualises the output.

Croston's method has been shown to be biased and various corrections have been pro-

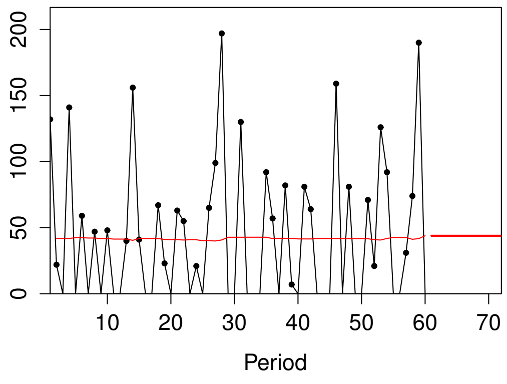


Figure 5.1: Croston's method fit and forecast.

posed, such as the Syntetos-Boylan-Approximation (SBA) (Syntetos and Boylan, 2005) and the Shale-Boylan-Johnston correction. These can be easily produced using the `crost` function.

```
# SBA
f.sba <- crost(y,h=h,type="sba")$frc.out
```

We may be interested in the Croston decomposition of a time series (for a more in-depth discussion of the Croston decomposition see Petropoulos et al., 2016). We can get this directly by using the function `crost.decomp`:

```
# Croston Decomposition
crost.decomp(y)
```

This provides the non-zero demand and the inter-demand intervals for the given series.

```
$demand
 [1] 132 22 141 59 47 48 40 156 41 67 23 63 55 21 65 99 197 130 92 57 82 7 81
     64 159 81 71 21 126 92 31 74 190

$interval
 [1] 1 1 2 2 2 2 3 1 1 3 1 2 1 2 2 1 1 3 4 1 2 1 2 1 4 2 3 1 1 1 3 1 1
```

Using this decomposition we can easily replace the exponential smoothing method used to forecast these two quantities with other ones. The function `crost.ma` replaces exponential smoothing with moving average.

```
# Moving Average Croston
f.cma <- crost.ma(y,h=h)$frc.out
```

A more recent alternative method is the TSB, which differs from Croston's method by updating modelling the probability of demand, instead of the inter-demand intervals and updating this in every period, instead of only when demand occurs as Croston's method prescribes. This allows it to better deal with obsolescence.

```
# TSB
f.tsb <- tsb(y,h=h)$frc.out
```

The *tsintermittent* package also contains a simple implementation of single exponential smoothing. Although it introduces decision bias for intermittent time series, it has been shown to still be a useful method.

```
# Single Exponential Smoothing
f.ses <- sexsm(y,h=h)$frc.out
```

5.3. Forecasting intermittent time series with temporal aggregation

An alternative way to deal with the problem of intermittence is to temporally aggregate the time series so that the degree of intermittence is minimised. The ADIDA method proposed by Nikolopoulos et al. (2011) is taking advantage of this idea. First it aggregates the series, then a forecast is produced at the aggregate level, which is in turn disaggregated to the original sampling frequency. However, a question posed by ADIDA to the modeller is what is the appropriate aggregation level. Nikolopoulos et al. (2011) recommend a simple heuristic that is the desired forecast horizon (lead time) plus one.

We can avoid choosing a single temporal aggregation level by implementing MAPA for intermittent time series. Although forecast combinations have been shown to perform poorly for intermittent demand, combinations over temporal aggregation levels have been found to improve accuracy (Petropoulos and Kourentzes, 2015). We can produce such forecasts using the function `imapa`.

```
# IMAPA
f.imapa <- imapa(y,h=h)$frc.out
```

If we restrict the function to use a single temporal aggregation level then we can get ADIDA based forecasts.

```
# ADIDA
f.adida <- as.vector(imapa(y,h=h,minimumAL=h+1,maximumAL=h+1)$frc.out)
```

5.4. Intermittent series classification

Both ADIDA and IMAPA as implemented in the *tsintermittent* package by default choose the appropriate method to use from Croston's, SBA and SES automatically, based on the PKa classification (Petropoulos and Kourentzes, 2015), which is an extension of the classification proposed by Syntetos et al. (2005), as refined by Kostenko and Hyndman (2006). All these classifications can be produced by the function `idclass`.

```
# The function can either handle a single series as a vector or multiple as
    an array.
```

```
# To use the single series we will first transform it into a vector.
idclass(as.vector(y),outplot="summary")
```

The resulting console output provides the classification outcome:

```
> idclass(as.vector(y),outplot="summary")
$idx.croston
integer(0)

$idx.sba
[1] 1

$idx.ses
integer(0)

$cv2
[1] 0.3859854

$p
[1] 1.787879

$summary
      Series
Croston   0
SBA       1
SES       0
```

To demonstrate how to classify multiple time series let us simulate some using the function `simID`. The options `type` and `outplot` allows to control the type of classification its visualisation. Figure 5.2 provides some examples.

5.5. Forecasting multiple time series

Forecasts for a complete dataset can be easily produced using the individual forecasts within a loop, or alternatively use the function `data.frc`. For example for 5 simulated series we can use:

```
# Forecasting Multiple Series
# For this example we produce 5 simulated series
f.data <- data.frc(simID(5,30),method="crost",type="sba",h=5)
# f.data stores all information for each series. We can simply get the
   forecasts for
# all series by typing
f.data$frc.out
```

The resulting forecasts are reported in the console:

```
> f.data$frc.out
      ts.1     ts.2     ts.3     ts.4     ts.5
1 23.48276 14.35443 22.90572 3.868408 56.70894
2 23.48276 14.35443 22.90572 3.868408 56.70894
```

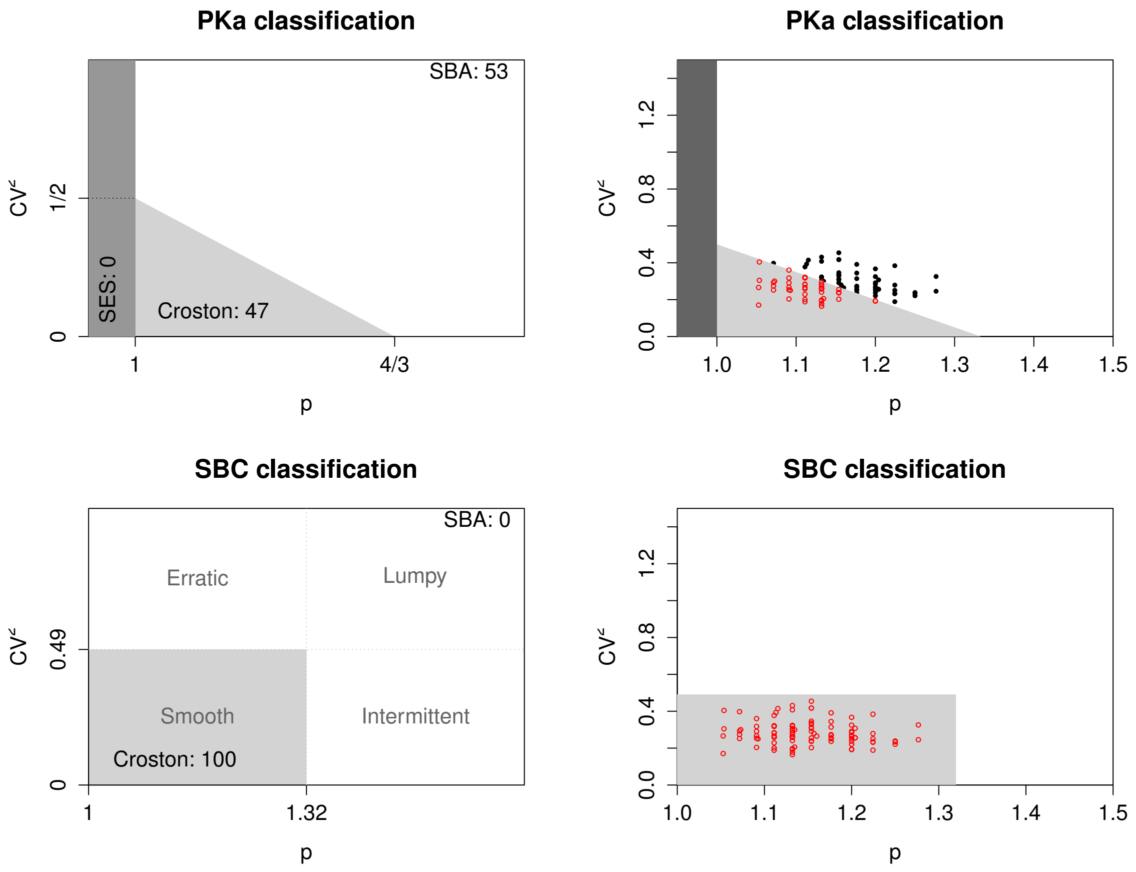


Figure 5.2: Examples of classification using `idclass`.

```
3 23.48276 14.35443 22.90572 3.868408 56.70894
4 23.48276 14.35443 22.90572 3.868408 56.70894
5 23.48276 14.35443 22.90572 3.868408 56.70894
```

Although in this example we pre-specified to use SBA for forecasting, we could have set `type="auto"`, which would choose for each time series the most appropriate method according to the PKa classification.

5.6. Forecast evaluation

Measuring forecasting performance for intermittent demand time series is an open research topic. The reader is referred to Kourentzes (2014) and Kolassa (2016). In this example though, we will content with ME and RMSE.

```
f.all <- cbind(f.crost,f.sba,f.cma,f.tsb,f.imapa,f.adida)
k <- dim(f.all)[2]
y.all <- t(tcrossprod(rep(1,k),y.test))
E <- y.all - f.all
ME <- apply(E,2,mean)
RMSE <- sqrt(apply(E^2,2,mean))
res <- rbind(ME,RMSE)
print(res)
```

This provides the following summary statistics, indicating that for the example time series (ts3) Croston's method is marginally better.

```
> print(res)
      f.crost  f.sba   f.cma   f.tsb f.imapa f.adida
ME   10.18332 10.26556 10.18816 10.81691 12.15261 11.41289
RMSE 93.02209 93.03113 93.02262 93.09358 93.25822 93.16471
```

6. Forecasting with causal methods

6.1. Functions

Table 6.1 lists the functions that we will be using in this section:

Table 6.1: Functions used for causal methods

Function	Package	Description
cor	stats	Calculate the correlation coefficient
lm	stats	Fit linear models
summary	stats	Produce result summaries of model fits
predict	stats	Provide predictions from model fits
acf	stats	Compute and plot the ACF function
hist	graphics	Compute and plot a histogram
dnorm	stats	Density function for the normal distribution

6.2. Simple regression

Sometimes it is useful to investigate relationships between two variables and subsequently to build regression models so that one variable can be used to predict another. For example, the sales of beer in a local pub could be heavily affected by external variables, such as the weather and/or sport events. In this section we will see how we can build such simple regression models using R.

In the first example, we assume that the sales of a company (contained in the file "salesreg.txt"[2]) are affected by the expenses occurred for advertising purposes. The advertising information is captured in the file "advertising.txt". First, we read the files using the function `scan()` and then we combine the two vector in a data frame. Subsequently, we produce a scatter-plot where the x-axis represents the advertising costs (independent variable) and the y-axis the sales (dependent variable). Scatter-plots allow us for investigating potential linear or non-linear correlations between two variables. The output is depicted in figure 6.1.

```
# Scan the sales and advertising data
sales <- scan("salesreg.txt")
advertising <- scan("advertising.txt")
# Combine the two set of data in a data frame
data <- data.frame(sales, advertising)
# Plot the data as a scatterplot
plot(advertising, sales, xlab="Advertising", ylab="Sales")
```

A statistical way to measure this relationship is the correlation coefficient, which in R can be computed with the function `cor()`.

[2]Remember to appropriately set the working directory.

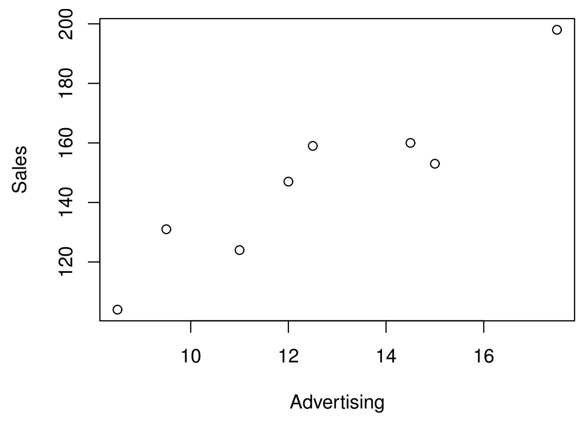


Figure 6.1: Scatter-plot of advertising costs against sales.

```
# Calculate the correlation
cor(advertising, sales)
```

The result in the console indicates a strong positive correlation between the two outputs. The positive strong correlation suggests that increase in advertising costs would result in increase in sales.

```
[1] 0.9302205
```

We now build a simple regression model of sales on advertising. The `lm()` function can be used for fitting linear models, where within the brackets we need to specify the relationship and the source for the data (data frame). The results summary of the fitted model can be displayed using the function `summary()`. The fitted values of the model can be displayed by typing `fit$fitted.values`. Finally, the function `lines()` is used to draw an additional line (the best fit line) on the existing graph.

```
# Fit a simple regression model
fit <- lm(sales ~ advertising, data)
# Show the summary of the fitted model
summary(fit)
# Draw the best fit line
lines(x=advertising, y=fit$fitted.values, col="red")
```

The output of the model summary is provided below. We can see that increase of one unit in advertising costs results in increase of almost 9 units in sales. The overall fit of the regression model is provided by the R^2 or coefficient of determination. In this case, we can see that 86.5% of the total variance has been captured by this simple regression model. Figure 6.2 presents the actual data (black) against the fitted values (red).

```
Call:
lm(formula = sales ~ advertising, data = data)
```

```
Residuals:
    Min      1Q  Median     3Q     Max
-15.3946 -7.8281 0.4656 8.4667 12.5486

Coefficients:
         Estimate Std. Error t value Pr(>|t|)
(Intercept) 36.735 18.196 2.019 0.090037 .
advertising 8.777   1.414 6.209 0.000806 ***
---
Signif. codes: 0 *** 0.001  **   0.01  *   0.05  .   0.1      1

Residual standard error: 11.2 on 6 degrees of freedom
Multiple R-squared: 0.8653, Adjusted R-squared: 0.8429
F-statistic: 38.55 on 1 and 6 DF, p-value: 0.000805
```

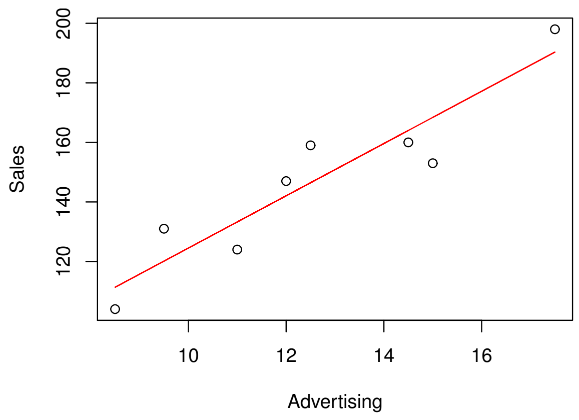


Figure 6.2: Best fit line (red) of the simple regression model against the actual observations (black).

Based on the fitted regression model, we are now in position to produce forecasts. Let us assume that the company wishes to investigate three scenarios regarding future advertising expenses, namely 13.8, 15.5 and 16.3. We first create a new data frame that contains the new values for the independent variable. Then we use the function `predict()`, where the fit of the model and the future values for the advertising costs are passed. By setting the argument `se.fit` equal to `TRUE` we additionally get information related to the standard error.

```
# Create a new data frame to hold advertising plans
new_data = data.frame(advertising=c(13.8, 15.5, 16.3))
# Calculate the predictions based on the fitted model, including the standard
    error
predict(fit, new_data, se.fit=TRUE)
```

The console output is:

```
$fit
      1       2       3
157.8619 172.7833 179.8051
```

```
$se.fit
       1        2        3
4.327773 5.737147 6.602084

$df
[1] 6

$residual.scale
[1] 11.19604
```

6.3. Linear regression on trend

Sometimes, independent variables (internal or external) are not available. However, regression modelling can still be performed on some very useful predictors, such as the trend indicator. In this example, we will model the quarterly sales of Apple iPhone on the trend. First, we load the respective data contained in the file "iphone.txt". Then, the data are transformed in a time series format and plotted. Finally, we define the trend indicatot (`t`) which is nothing more than the indication of the time stamp (1, 2, 3, ...). This indicator is useful when a linear trend exists.

```
# Scan the quarterly sales of the iPhone and transform into a time series
iphone <- ts(scan("iphone.txt"), start=c(2007,2), frequency=4)
# Plot the data
plot(iphone, ylab="Sales", main="iPhone quarterly sales")
# Create a trend indicator
t = 1:length(iphone)
```

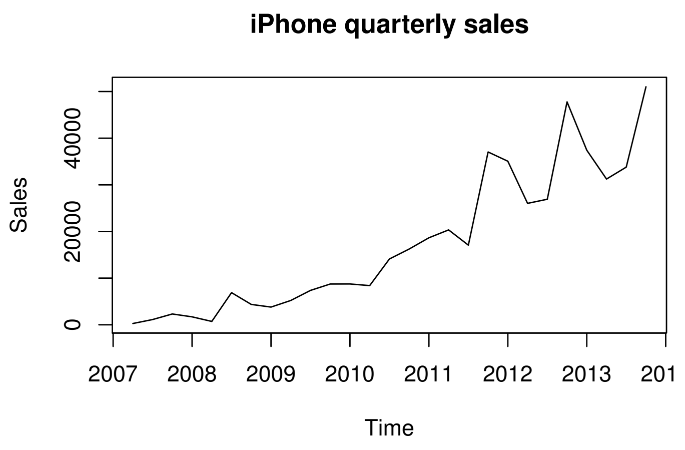


Figure 6.3: Quarterly sales of the Apple iPhone (in thousands).

As previously, the linear regression model can be estimated from the function `lm`.

```
# Fit a simple regression model
```

```
fit <- lm(iphone ~ t)
# Show the summary of the fitted model
summary(fit)
# Add to the existing graph the fitting values
lines(ts(fit$fitted, start=c(2007,2), frequency=4), col="red", lwd=2)
```

The output in the console is presented below. Overall, this appears to be a good model. However, residual analysis and diagnostics will help us in identifying specific problems.

```
Call:
lm(formula = iphone ~ t)

Residuals:
   Min     1Q Median   3Q   Max
 -7530 -3666 -2488 3991 14300

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -7385    2354 -3.137 0.00434 **
t            1777     147 12.093 6.08e-12 ***
---
Signif. codes: 0 *** 0.001 **  0.01 *  0.05 .  0.1     1

Residual standard error: 5947 on 25 degrees of freedom
Multiple R-squared: 0.854, Adjusted R-squared: 0.8482
F-statistic: 146.2 on 1 and 25 DF, p-value: 6.084e-12
```

6.4. Residual diagnostics

First, we plot the residuals of the fit (`fit$residuals`) against time. We would expect not to find any specific patterns and the residuals to be stationary in nature. However, we can see in figure 6.4 that the residuals consistently have negative values for the periods 7 up to 18, while the variance seems to increase after the 19^{th} period. These suggest that a non-linear patters between sales and trend possibly exists.

```
# Plot the residuals on time
plot(fit$residuals, ylab="Residuals", xlab="Year")
abline(h=0, col="grey")
```

Then, we compute and plot the auto-correlation function plot using the R function `acf()`. The output (figure 6.5) suggests that there is a significant positive value for lag equal to 4. Given that our data are quarterly, this could be an indication of a seasonal patterns. One solution would be to add indicator for the seasonality (seasonal dummies). However, a "market intelligence" view on the sales graph (figure 6.3) suggests that the peaks in the sales are associated with the releases of new versions of the Apple iPhone (which also possibly have lead effects, as customers would anticipate that a new version will be released). As such, a better way to model this series would be to include a dummy variable for new releases.

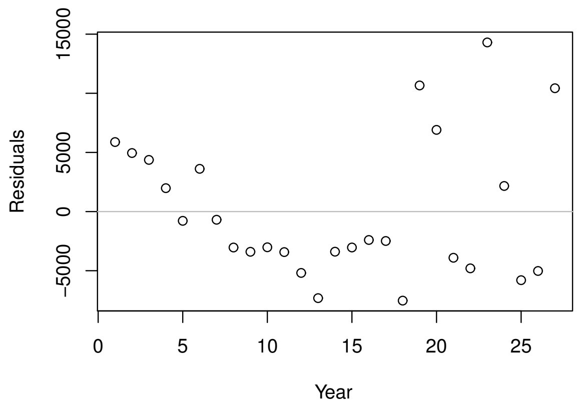


Figure 6.4: Plot of the residuals on time.

```
# Check for autocorrelation in the residuals
acf(fit$residuals)
```

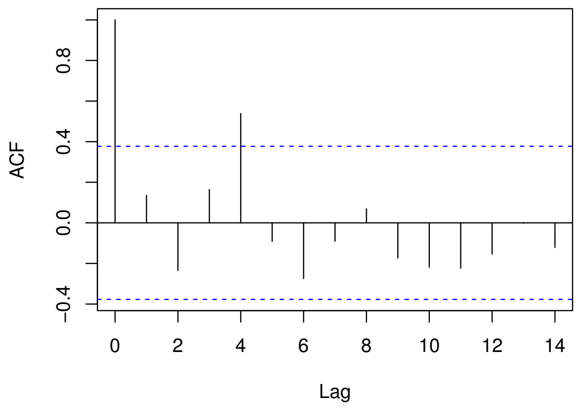


Figure 6.5: ACF plot of the residuals.

Finally, we produce a histogram of the residuals to check if these are normally distributed. The R function `hist()` computes and plots a histogram, where the argument `prob` controls if a relative or absolute frequency histogram will be computed. We also draw a line of the expected normal distribution density function given the standard deviation of the residuals. A comparison of the actual histogram and the theoretically expected values suggests that the residuals are not normally distributed but are positively skewed.

```
# Histogram of residuals
hist(fit$residuals, xlab="Residuals", prob=TRUE)
# Compare with normal distribution
lines(-10000:15000, dnorm(-10000:15000, 0, sd(fit$residuals)), col="red")
```

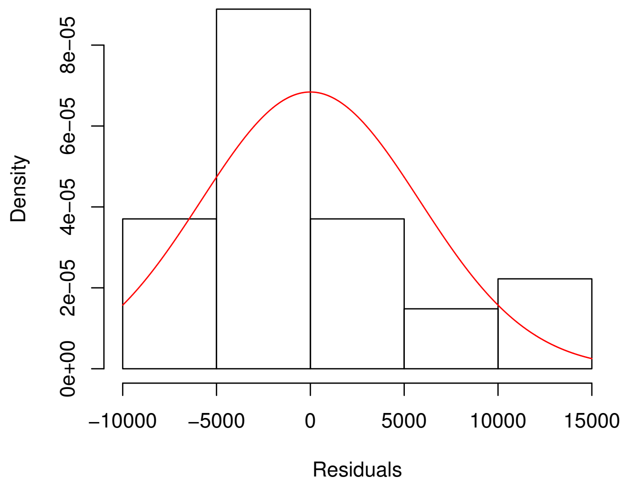


Figure 6.6: Density histogram of the residuals against theoretically expected values.

6.5. Multiple regression

Multiple regression is useful when we would like to model the impact of two or more independent variables (also called regressors) on the dependent variable (sales or demand or ...). As we saw in the previous section, the residual analysis of the model referring to the sales of the Apple iPhone indicated that, apart from the trend indicator, we should additional consider other variables, such as new product launches. Other variables that could be generally considered in a multiple regression model include seasonal dummy variables as well as other dummy variables so that the impact of special events and actions (promotions, strikes, holidays, etc) is captured.

In this example we will model and forecast the weekly sales of a company using as regressors information regarding promotional activity. Assuming that three types of promotions take place, we will introduce three independent dummy variables. Each of thee variables will have value 1 if a promotion took place at the respective period, o otherwise.

First, let us read and plot the data, which are contained in the data file "sales.txt". We will also define a new variable (`n`) where the length of the in-sample data is saved.

```r
# Scan data (sales)
sales <- scan("sales.txt")
# Plot the sales as a line
plot(sales, type="l", xlab="Weeks", ylab="Sales")
# Save the length of the sales into a variable
n <- length(sales)
```

Now, we need to read the promotional information which are contained in three external data files, "promo1.txt", "promo2.txt" and "promo3.txt". We scan and arrange the data into a data frame. Note that each of these series holds more data points than the available in-sample (sales) data; this is because the company has information regarding forthcoming (planned) promotional activity for the next 13 weeks. We define as the forecast horizon (`h`) the length of this additional information.

```
# Scan the promotional data and save into a data frame
promos <- data.frame(promo1=scan("promo1.txt"), promo2=scan("promo2.txt"),
    promo3=scan("promo3.txt"))
# Diplay the data frame in the console
promos
# Save the length of the information regarding forthcoming promotional
    activity (or forecasting horizon)
h <- nrow(promos)-n
```

The console output is:

```
   promo1 promo2 promo3
1       0      0      0
2       0      0      0
3       0      0      0
4       0      0      1
5       0      0      1
6       0      0      0
7       0      0      0
8       0      1      1
9       0      0      1
10      0      0      1
...
```

The promotional activity information can be graphically displayed on the sales graph, so that we can qualitatively assess the uplift in sales given specific types of promotion. We can add additional points on the existing plot using the `points()` function. The `which()` function is also useful so that we are able to distinguish the periods with promotional activity. As different promotional types are expected to have different impact on the sales, we present this information using different markers (pch argument) and different colours (`col` argument). Lastly, the `cex` argument controls the size of the markers.

Figure 6.7 presents the weekly sales information and the periods with promotional activity.

```
# Plot the promotional activity information on the existing plot
points(which(promos[,1]==1), sales[which(promos[,1]==1)], pch=1, col="blue",
    cex=1.5, lwd=2)
points(which(promos[,2]==1), sales[which(promos[,2]==1)], pch=2, col="red",
    cex=1.5)
points(which(promos[,3]==1), sales[which(promos[,3]==1)], pch=3,
    col="orange", cex=1)
```

We will now build our first multiple regression model. We will use the function for linear models (`lm()`), where the first argument gives the relationship of the variables (sales depended on promotions) and the second argument provides the data frame that contains the information for the independent variables. Note that only the first `n` observations of `promos` can be used (as this is also the length of the available sales information). The

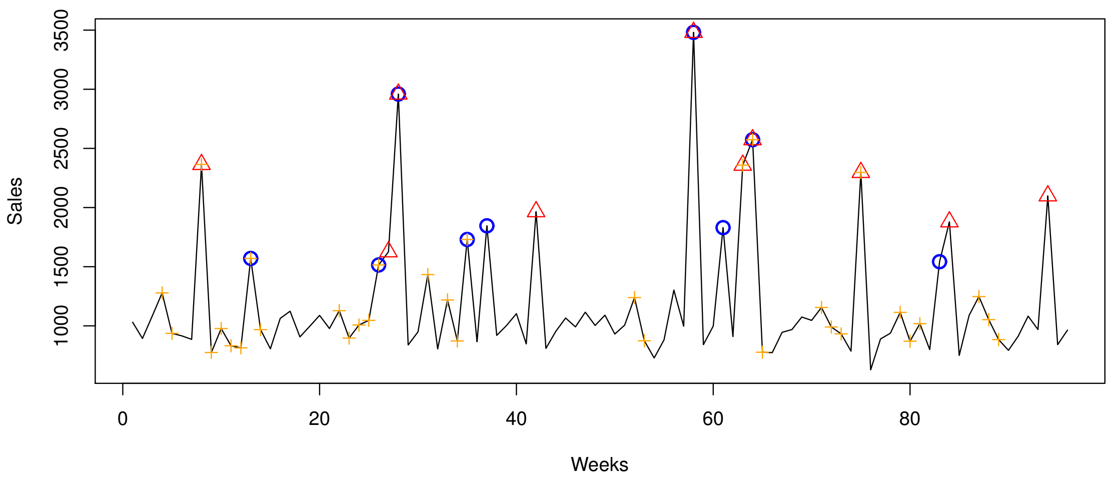


Figure 6.7: Visualisation of the weekly sales and the periods with promotional activity.

function `summary()` returns the usual output of the regression model. Finally, we add the fitted values on the existing graph (figure 6.8).

```
# Fit an additive regression model
fit1 <- lm(sales ~ promo1 + promo2 + promo3, promos[1:n,])
# Return the summary of the model
summary(fit1)
# Add a new line (the fit of the model) on the existing graph
lines(fit1$fit, col="blue")
```

The console output for the summary of the fit is:

```
Call:
lm(formula = sales ~ promo1 + promo2 + promo3, data = promos[1:n, ])

Residuals:
   Min     1Q Median    3Q    Max
-483.44 -120.07 -7.94 120.73 613.04

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  943.13  22.18 42.516 <2e-16 ***
promo1     758.52    60.92 12.450 <2e-16 ***
promo2    1166.43    58.05 20.095 <2e-16 ***
promo3      58.30    36.31 1.605  0.112
---
Signif. codes: 0 *** 0.001  **   0.01  *   0.05  .   0.1      1

Residual standard error: 168.5 on 92 degrees of freedom
Multiple R-squared: 0.8882, Adjusted R-squared: 0.8845
```

```
F-statistic: 243.6 on 3 and 92 DF, p-value: < 2.2e-16
```

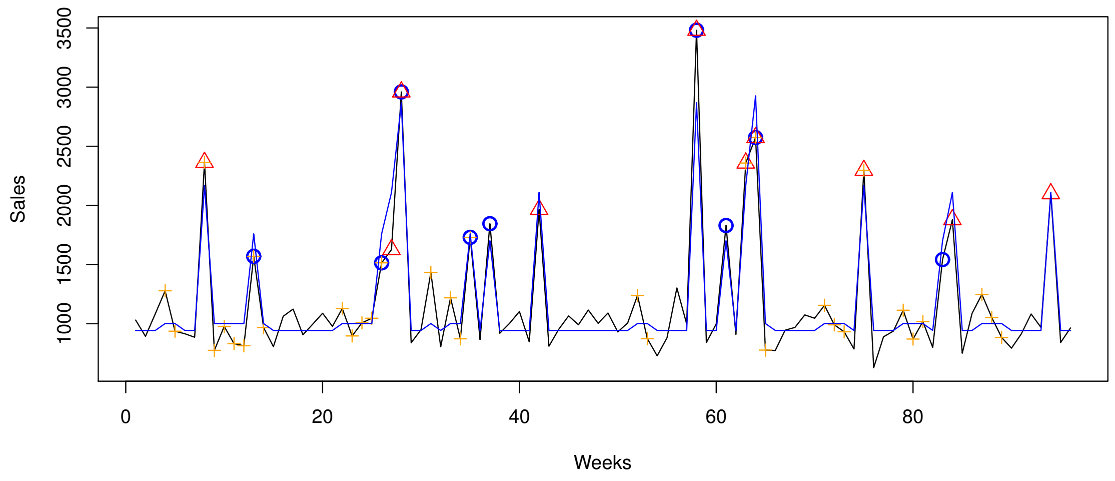


Figure 6.8: Graph of the linear regression fit on the actual data.

Figure 6.8 makes apparent that there is a significant negative lagging effect from the promotional activity: the sales significantly decrease in the periods after a promotion has occurred. However, this is not captured in our model (blue line). In order to be able to model this lagging effect, we need to consider lags of the independent variables.

Let us create some lags; there are many ways to go about doing this, we will show one of the simplest here. The variables we want to lag are contained in `promos`. Assuming that no promotional activity occurred in period 0, we first create a vector of three zeros (0, 0, 0), i.e. zero indicator for the first observation that we do not have for the lags. Then, we paste this first observation on top of `promos` (using the `rbind()` function), effectively shifting all by one period in the past. Lastly, we change the names of the three new variables and update the `promos` so that this now includes non-lagged and lagged promotional information.

```
# Create lagged variables of the promotions
promos_lag <- rbind(c(0,0,0), promos)
names(promos_lag) <- c("promo1_lag", "promo2_lag", "promo3_lag")
# Combine the two data frames
promos <- cbind(promos, promos_lag[1:(n+h),])
# Diplay the expanded data frame in the console
promos
```

The console output is:

```
    promo1 promo2 promo3 promo1_lag promo2_lag promo3_lag
1        0      0      0          0          0          0
2        0      0      0          0          0          0
```

```
3        0      0      0        0        0        0
4        0      0      1        0        0        0
5        0      0      1        0        0        1
6        0      0      0        0        0        1
7        0      0      0        0        0        0
8        0      1      1        0        0        0
9        0      0      1        0        1        1
10       0      0      1        0        0        1
...
```

We build another additive multiple regression model that includes six independent variables (non-lagged and lagged promotional activity).

```
# Fit an additive regression model, adding the lagged effects of the
   promotions
fit2 <- lm(sales ~ promo1 + promo2 + promo3 + promo1_lag + promo2_lag +
   promo3_lag, promos[1:n,])
# Return the summary of the model
summary(fit2)
# Add a new line (the fit of the model) on the existing graph
lines(fit2$fit, col="green")
```

The summary of the new model is:

```
Call:
lm(formula = sales ~ promo1 + promo2 + promo3 + promo1_lag +
   promo2_lag + promo3_lag, data = promos[1:n, ])

Residuals:
    Min     1Q Median    3Q    Max
-263.72 -60.76 -7.54 54.53 540.79

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1012.32 16.59 61.037 < 2e-16 ***
promo1     793.74    42.39 18.723 < 2e-16 ***
promo2    1134.28    40.67 27.892 < 2e-16 ***
promo3     176.56    28.79 6.134 2.32e-08 ***
promo1_lag -71.48    42.72 -1.673 0.097799 .
promo2_lag -148.01  40.00 -3.700 0.000373 ***
promo3_lag -255.06  29.04 -8.783 1.06e-13 ***
---
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 114.6 on 89 degrees of freedom
Multiple R-squared: 0.95, Adjusted R-squared: 0.9466
F-statistic: 281.9 on 6 and 89 DF, p-value: < 2.2e-16
```

The second model has a higher adjusted R^2 value. However, the lagged variable for the first type of promotion appears to be insignificant at level 0.05, so we could also remove it from the model. Additionally, and similarly to creating one-period lags, one could consider creating and adding in the model lags (or leads) of the promotional data for more periods.

Let us assume that the `fit2` is the model that we choose as the best one. Now, we will use this model and the information regarding forthcoming promotional activity and we will produce forecasts for the next 13 weeks. For this purpose, we use the function `predict()`, where the rows `n+1` up to `n+h` of the data frame `promos` are used as future data.

```
# Calculate the out-of-sample forecasts, based on the available information
   on forthcoming promotions
fcs <- predict(fit2, promos[(n+1):(n+h),])
fcs
```

We can plot the sales and the forecasts together. We will need to specify the limits for the horizontal axis (time) so that there is enough space for the 13-steps-ahead forecasts to be drawn. Figure 6.9 presents the output.

```
# Plot the sales as a line
plot(sales, type="l", xlab="Weeks", ylab="Sales", xlim=c(1,109))
# Plot the forecasts as a new line
lines(x=(n+1):(n+h), fcs, col="blue")
```

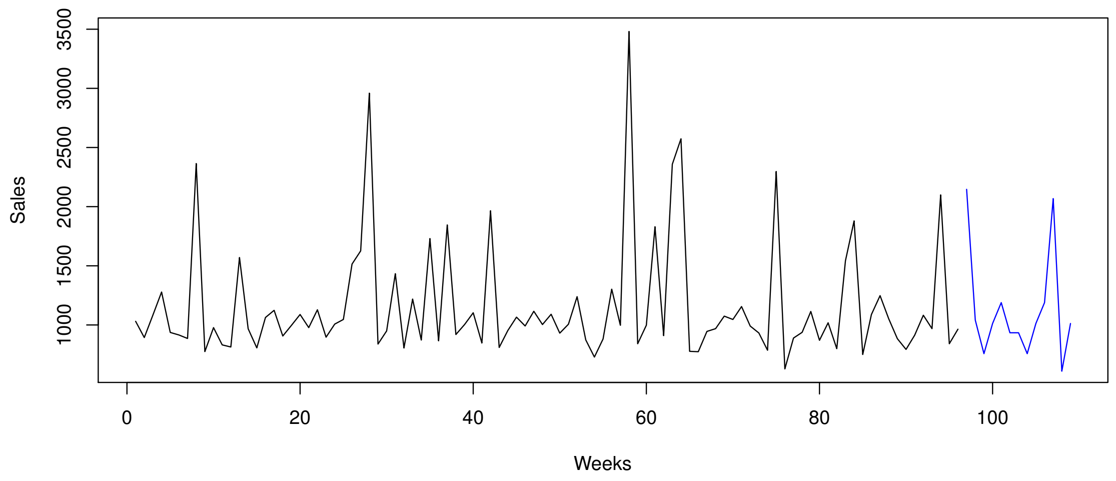


Figure 6.9: Line graph of the sales data and the 13-weeks-ahead forecasts from the additive regression model.

We will now build a multiplicative regression model. To do so, we need to transform our variables in logs (apart from the dummies). As with other programming languages, the `log()` function returns the natural logarithm on a positive number. Once we fit the new model (based on the logged sales), we need transform the fitted values using the `exp()` function, which computes the exponent.

```
# Calculate the natural logarithm of the sales
logsales <- log(sales)
# Fit a regression model
fitlog <- lm(logsales ~ promo1 + promo2 + promo3 + promo1_lag + promo2_lag +
    promo3_lag, promos[1:n,])
# Return the summary of the model
summary(fitlog)
# Plot the sales as a line
plot(sales, type="l", lwd=2)
# Add a line for the fitted values (which are transformed using the
    exponential function)
lines(exp(fitlog$fit), col="orange")
```

Figure 6.10 presents the graph of the sales and the fitted values from the multiplicative regression model.

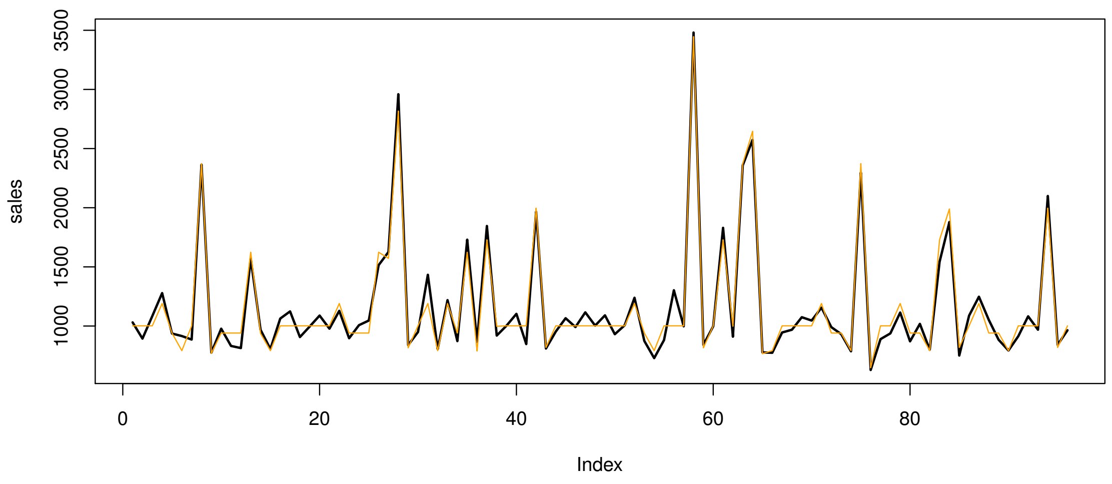


Figure 6.10: Line graph of the sales data and the fitted values from the multiplicative regression model.

6.6. Selecting independent variables

In multiple regression, it is important to be able to select the best subset of independent variables. One way to do this is to build all possible multiple regression models, evaluate them and select the best one. However, this approach is not practical when the number of predictors increases. For example, 10 regressors result in $2^{10} = 1024$ possible models.

In such cases, the backwards stepwise regression is regarded as a useful approach. The search for the optimal model starts from the assumption that the most complex model (the one that contains all available independent variables) is the best one. Then, one variable is removed at a time and the performance of the new model is recalculated. If a better model is found, then this will replace the best one. The procedure is repeated until no better model can be found.

An R implementation of the backwards stepwise regression that utilises the Akakie's Information Criterion (AIC) to measure the accuracy of each fit is presented below. Note that this example uses the same data from subsection 6.5.

```
# Backwards stepwise regression
step(lm(sales ~ promo1 + promo2 + promo3 + promo1_lag + promo2_lag +
   promo3_lag, promos[1:n,]))
```

The console output is presented below. We can see that the model containing all six variables has an AIC equal to 917.06, while when removing one variable at a time the result is a worse model (higher values of AIC). Thus, in this case the best model is the one containing all six variables.

```
Start: AIC=917.06
sales ~ promo1 + promo2 + promo3 + promo1_lag + promo2_lag + promo3_lag

          Df Sum of Sq   RSS     AIC
<none>                 1168490  917.06
- promo1_lag 1  36756  1205246  918.03
- promo2_lag 1 179758  1348247  928.80
- promo3   1    493934  1662424  948.91
- promo3_lag 1 1012860 2181349  974.99
- promo1   1   4602607  5771096 1068.39
- promo2   1  10214283 11382772 1133.59

Call:
lm(formula = sales ~ promo1 + promo2 + promo3 + promo1_lag +
   promo2_lag + promo3_lag, data = promos[1:n, ])

Coefficients:
(Intercept)    promo1     promo2     promo3 promo1_lag promo2_lag promo3_lag
   1012.32    793.74    1134.28    176.56     -71.48    -148.01    -255.06
```

7. Advanced methods in forecasting

In this section we will be showing how to do in R some specialised forecasting tasks. Each subsection is independent and uses a specific R package.

7.1. Hierarchical forecasting

Usually, a company's data can be naturally organised in multilevel hierarchical structures. However, forecasting is challenging as forecasts derived from different levels do not typically sum up. For example, the sum of the bottom-level forecasts is not equal to the forecasts of the top (company) level. In this section we will present how hierarchical reconciliation approaches can be applied in R. Such approaches include the bottom-up, top-down, middle-out and optimal combination. For more details on the hierarchical reconciliation approaches, you can see Hyndman et al. (2011) and Athanasopoulos et al. (2009).

For this section we will be using the *hts* and *fpp* packages. We can load the packages by typing:

```
# Load the requires packages
library(hts)
library(fpp)
```

The first package contains all the relevant functions for hierarchical forecasting and reconciliation, while the second package will provide some cross-sectional data suitable for hierarchical forecasting. In more detail, the data refer to the Australian tourism demand data and are adopted from the textbook Hyndman and Athanasopoulos (2013), example 9.7. The hierarchy of the data is depicted on figure 7.1. The bottom level data are contained in the variable `vn` of the *fpp* package.

Table 7.1 lists the functions that we will be using for hierarchical forecasting:

Table 7.1: Functions used for hierarchical forecasting

Function	Package	Description
hts	hts	Create hierarchical time series
forecast.gts	hts	Methods for forecasting hierarchical or grouped time series
allts	hts	Extract all time series from a gts object

First, let us have a look at the available data:

```
# Bottom level data
vn
# Return the first time series
vn[,1]
```

The data can be organised in a hierarchy using the function `hts`, where the `nodes` argument consists of a list that corresponds with how nodes are connected across the hierarchical levels. Once the hierarchy is saved in the new variable `y`, we can call for the names of the nodes and the connections (`y$labels` and `y$nodes` respectively).

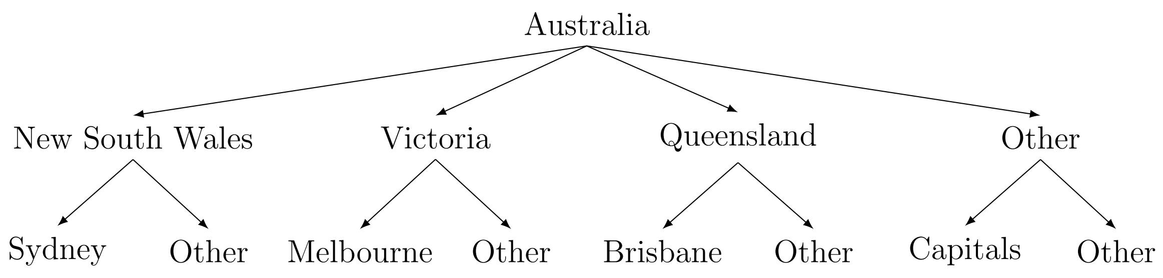


Figure 7.1: Hierarchical structure of Australian tourism demand data.

```
# Convert the data into an hierarchy
y <- hts(vn, nodes=list(4, c(2,2,2,2)))
# Return the hierarchy
y
# Return the labels of the nodes at each level
y$labels
# Return how the nodes are organised
y$nodes
```

The output in the console is:

```
Hierarchical Time Series
3 Levels
Number of nodes at each level: 1 4 8
Total number of series: 13
Number of observations per series: 56
Top level series:
      Qtr1  Qtr2  Qtr3  Qtr4
1998 84502 65314 72749 70891
1999 86891 66872 72179 68323
2000 85650 64467 70408 72859
...

$`Level 0`
[1] "Total"
$`Level 1`
[1] "A" "B" "C" "D"
$`Level 2`
[1] "Sydney" "NSW"     "Melbourne" "VIC"   "BrisbaneGC"
[6] "QLD"    "Capitals" "Other"

$`Level 1`
[1] 4
$`Level 2`
[1] 2 2 2 2
```

Once the data are organised within a hierarchy, we can directly produce forecasts using the `forecast.gts` function. Arguments of this function include the required forecast horizon (`h`), the reconciliation method (`method`) and the forecasting method (`fmethod`). Setting the horizon equal with 4 quarters ahead, the reconciliation method being the bottom-up approach and the exponential smoothing (ets) as the forecasting method, we have:

```
# Bottom-up approach
allf <- forecast.gts(y, h=4, method="bu", fmethod="ets")
# Summary of the output
allf
# All-levels forecasts
allts(allf)
# Bottom-level forecasts
allf$bts
# Plot all forecasts
plot(allf)
```

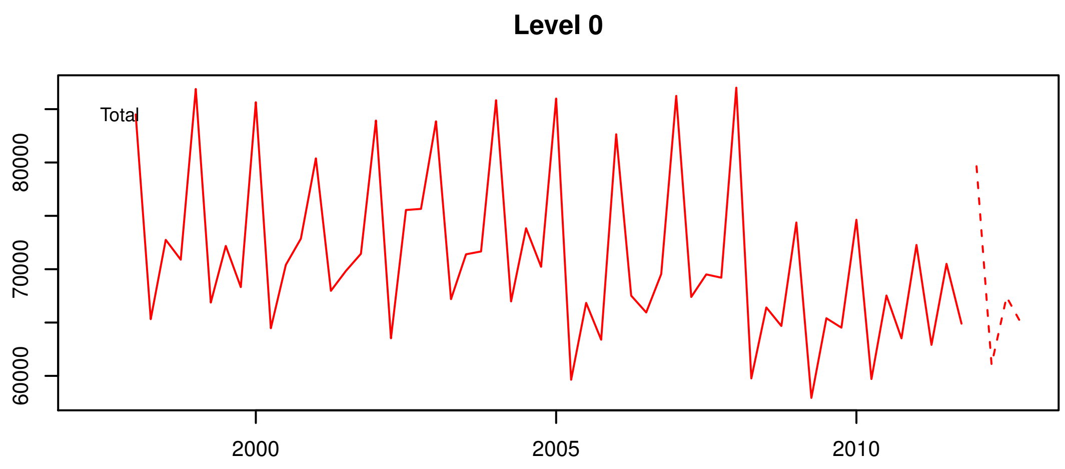
In this example, only the bottom-level forecasts are taken into account, while forecasts of higher levels are simply produced as the sum of the respective bottom-level forecasts. The output in the console is:

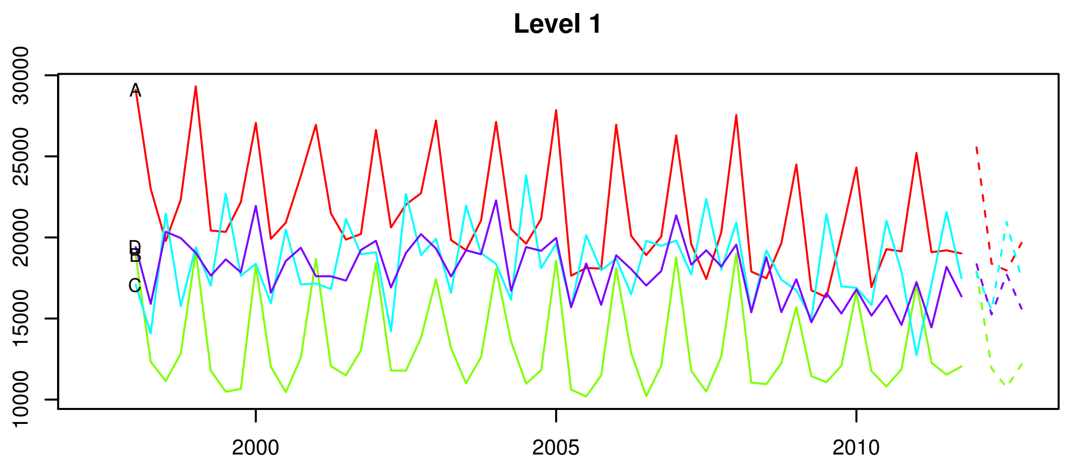
```
Hierarchical Time Series
3 Levels
Number of nodes at each level: 1 4 8
Total number of series: 13
Number of observations in each historical series: 56
Number of forecasts per series: 4
Top level series of forecasts:
         Qtr1     Qtr2     Qtr3     Qtr4
2012 79650.72 61059.14 67400.77 64847.41

           Total        A        B        C        D    Sydney      NSW ...
2012 Q1 79650.72 25558.82 17898.63 17841.70 18351.57 5883.261 19675.56
2012 Q2 61059.14 18384.19 11927.79 15498.77 15248.40 4938.497 13445.69
...
```

Figure 7.2 presents the output (data and reconciled forecasts across all levels) in a graphical format.

Other reconciliation approaches can be applied by suitably changing the value of the argument `method`. Note that there are three different top-down approaches, depending on how disaggregation is applied (i.e. how the proportions are calculated). Also note that the optimal combination approach requires (compared to the rest of the approaches) the highest computational time, as forecasts are independently produced at every node of the hierarchy.

```
# Top-down approach (Average historical proportions)
# for proportion of historical averages, use "tdgsf"
# for forecast proportions, use "tdfp"
allf <- forecast.gts(y, h=4, method="tdgsa", fmethod="ets")
plot(allf)
```





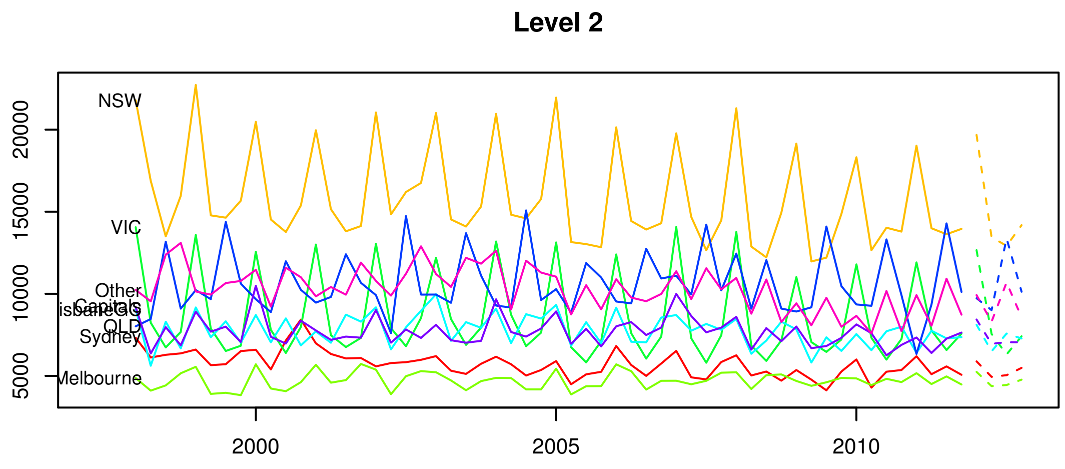


Figure 7.2: Hierarchical time series of the Australian domestic tourism and 4-step-ahead reconciled forecasts using the bottom-up approach and ETS.

```
# Middle-out approach
allf <- forecast.gts(y, h=4, method="mo", fmethod="ets", level=1)
plot(allf)

# Optimal reconciliation approach
allf <- forecast.gts(y, h=4, method="comb", fmethod="ets")
plot(allf)
```

Other forecasting methods that could be alternatively applied using the `forecast.gts` function include the ARIMA models and the random walk (where the `fmethod` argument should be set to "arima" or "rw" respectively).

7.2. ABC-XYZ analysis

Another interesting analysis for the forecasting process is the ABC-XYZ analysis. This analysis attempts to classify time series according to their importance and forecastability, so as to help prioritising the effort put in forecasting, or help exploring forecast accuracy in more detail. Both ABC (importance) and XYZ (forecastability) analyses are based on the Pareto analysis.

The *TStools* package offers three functions to help us perform these analyses, as seen in table 7.2

Table 7.2: Functions used for causal methods

Function	Package	Description
abc	TStools	Perform ABC analysis
xyz	TStools	Perform XYZ analysis
abcxyz	TStools	Combined results of ABC-XYZ

For this example we will load the necessary packages and some quarterly time series from the M3 competition to analyse:

```
# load the necessary packages
library(Mcomp)
library(TStools)

# Get 100 quarterly time series
M3.subset <- subset(M3, "QUARTERLY")
x <- matrix(NA,46,100)
for (i in 1:100){
  x.temp <- c(M3.subset[[i]]$x,M3.subset[[i]]$xx)
  x[1:length(x.temp),i] <-x.temp
}
```

To perform ABC analysis use the `abc` function. Unless the importance of a series is calculated externally, it is estimated as the mean volume of sales of a product. The function allows us to control how many classes we want and what percentages are assigned to each one through the argument `prc`. The default option is `prc=c(0.2,0.3,0.5)`, which means that three classes will be calculated with percentages 20, 30 and 50% respectively.

```
# ABC analysis
imp <- abc(x,outplot=1)
print(imp)
```

The option outplot provides the output in figure 7.3, while the console output of the analysis is:

```
> print(imp)
$value
  [1] 4775.589 2792.300 3442.898 3728.123 2546.471 8055.568 7813.196 4677.895
      4336.207 5144.083 4802.705 5835.723 4687.159 2633.124 2622.227 4618.660
      4526.615 3175.285 4781.364 3699.098 3827.364 5056.830 4284.986 4163.822
      4798.509 2596.903
 ...
 [92] 5690.931 6680.239 3118.496 4015.984 4654.409 6969.290 6211.645 4764.468
     3349.105

$class
    [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
[1,] "A"  "A"  "C"  "C"  "C"  "C"  "C"  "B"  "C"  "C"   "B"   "C"   "C"
...
    [,91] [,92] [,93] [,94] [,95] [,96] [,97] [,98] [,99] [,100]
[1,] "A"   "C"   "A"   "A"   "C"   "B"   "A"   "B"   "C"   "C"

$rank
  [1]  6  7 62 75 97 61 93 40 84 67 36 55 98 58 65 83 12 63 92 33 69 77 32 59
      46 29 27 71 89 74 10 47 22 53 49 43 48 11 25 19 1 99 13  8 52 96 16 37
      17 80  9 82 23 66 41 70 24 35 72 68
 ...
 [91]  2 87 14 15 78 26  5 42 54 88

$importance
     [,1]
A 28.68601
B 33.06462
C 38.24938
```

Figure 7.3 shows how concentrated is the value in the A, B or C classes. The bigger the difference between the straight diagonal line and the concentration curve, the more value is included in the A and B classes. For example, here we have set class A to contain 20% of the products, but it accounts for 28.7% of the value.

Similarly XYZ analysis can be done using the `xyz` function. Forecastability can be estimated in three different ways using this function. The default is to calculate a scaled in-sample RMSE of the best of two methods: Naive and Seasonal Naive. The scaling is done by dividing the RMSE by the mean of each time series. The second approach is to calculate the coefficient of variation for each time series. The last approach is to fit an ETS model (using the *forecast* package) and calculate again the scaled in-sample RMSE. The advantage of the first approach is that it can tackle with seasonal time series and it is relatively fast.

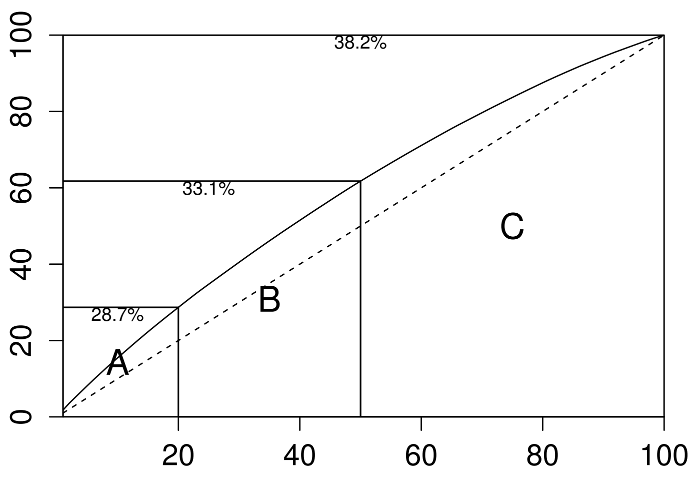


Figure 7.3: ABC analysis.

The second approach is very simple, but it does not handle well trended or seasonal time series. The last approach selects the appropriate ETS model for each time series, thus being able to handle most types of time series. On the downside, it requires substantially more time.

```
# XYZ analysis
frc <- xyz(x,m=4,outplot=TRUE)
```

The visual output of the XYZ analysis is given in figure 7.4. As with the `abc` function, external measures of forecastability can be used and any number of classes with different percentages.

Finally we can use `abcxyz` to get the combined classification:

```
# ABC-XYZ
abcxyz(imp,frc)
```

This provides a summarised console output and a combined visualisation, as in figure 7.5.

```
> abcxyz(imp,frc)
   Z  Y  X
A  4  7  9
B  6  7 17
C 10 16 24
```

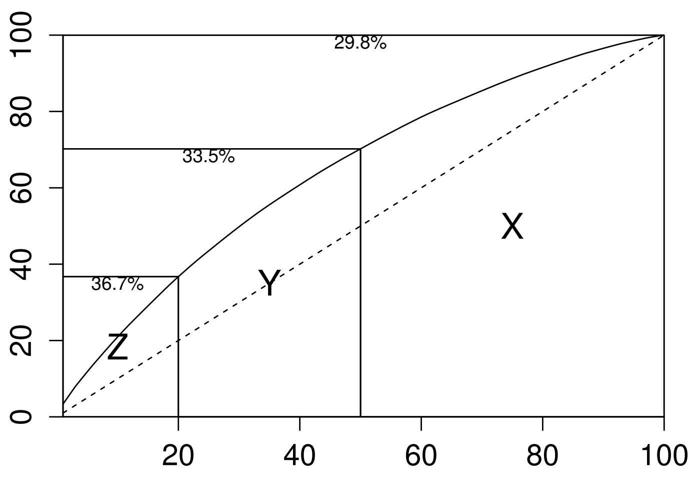


Figure 7.4: XYZ analysis.

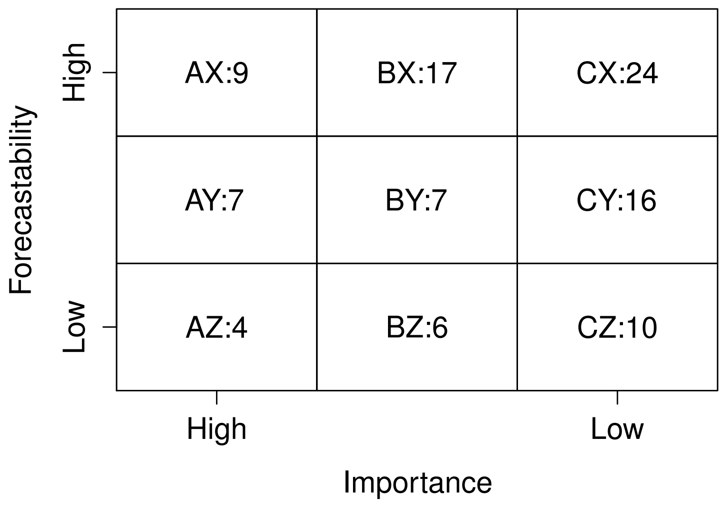


Figure 7.5: ABC-XYZ analysis.

7.3. A modern take on regression: LASSO

Recently there has been a lot of interest in shrinkage methods. Least Absolute Shrinkage and Selection Operator (LASSO) regression has gained a lot of popularity, mainly because it deals with several problems that conventional stepwise regression fails. Briefly shirnkage methods, and in particular LASSO, have been shown to: (i) be able to select reasonable variables even when the number of explanatory variables exceeds the available data points;

(ii) be very robust against multicollinearity; (iii) be robust against sample uncertainty. The reader is referred to Friedman et al. (2001) for an excellent introduction to the topic.

The package *glmnet* has several helpful functions to use LASSO, amongst other shrinkage regression methods. To demonstrate its use we will revisit an example from section 6.2. The example dataset has only 8 data points and we used a single predictor. Building models with so few observations is risky, but it becomes impossible if the number of potential explanatory variables exceeds the number of observations. This is one of the areas that LASSO can demonstrate its strength. For an example on how conventional selection methods fails once the regression is saturated with input variables, you can check how to fit an elephant!

We will use the functions listed in table 7.3 from the *glmnet* package for this example.

Table 7.3: Functions used for LASSO regression

Function	Package	Description
glmnet	glmnet	Fit LASSO regression
cv.glmnet	glmnet	Fit LASSO regression and find CV optimal λ
coef	glmnet	Extract regression coefficient from LASSO fit

First let us load the necessary package and some data:

```
library(glmnet)

# Let us get some data from the causal modelling examples
sales <- scan("salesreg.txt")
advertising <- scan("advertising.txt")

# This example has 8 data points, which is already too little
# Suppose instead of one regressor we had 22. Conventional regression
# will fail in this case.

# Let us load some additional potential explanatory variables
X <- read.csv("indicators.csv")

# Combine all data sets in a data frame
data <- data.frame(sales, advertising,X)
```

Let us check the correlation between the target sales variable and all other.

```
> round(cor(data)[1,],2)
      sales advertising Ind_898 Ind_1934
       1.00        0.93   -0.34    -0.30
   Ind_3184 Ind_4399 Ind_4513 Ind_6092
      -0.22    -0.32     0.28    -0.05
    Ind_901 Ind_5880 Ind_19353 Ind_19417
      -0.33     0.32    -0.38    -0.28
  Ind_25718 Ind_25719 Ind_25884 Ind_25705
      -0.24     -0.42     -0.14     -0.15
  Ind_25707 Ind_25714 Ind_25716 Ind_25538
      -0.34     -0.36     -0.40     -0.28
```

```
 Ind_25540 Ind_25547 Ind_25549
     -0.52     -0.52     -0.13
```

Advertising remains the most correlated variable, but there are other potentially useful ones as well. As noted, stepwise regression cannot solve this problem due to the number of variables. Let us attempt to solve this will LASSO regression. LASSO operates by minimising the following cost function: $MSE + \lambda \sum (|\mathbf{b}|)$, where $\mathbf{b}$ are the coefficients of the explanatory variables and λ controls the amount of shrinkage.

```
# Fit LASSO regression
fit <- glmnet(x=as.matrix(data[,2:23]),y=data[,1])
# This gives us the optimal set of coefficients for each lambda. We can plot
   this
plot(fit,xvar="lambda")
```

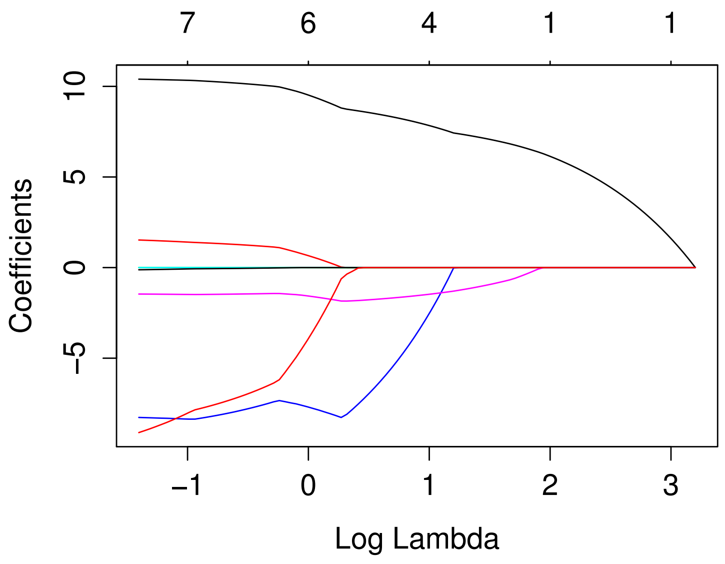


Figure 7.6: Coefficients for different λ.

Figure 7.6 presents the resulting output and shows that as λ increases more variables are removed from the regression. Selecting the best λ is not trivial. One approach to resolve this is to use cross-validation. We can do this by using the `cv.glmnet` function.

```
# Cross-validated LASSO fit
fit.cv <- cv.glmnet(x=as.matrix(data[,2:23]),y=data[,1],grouped=FALSE)
# Let us get the coefficients from this fit
beta.lasso <- coef(fit.cv)
```

The resulting coefficients are reported in the console output:

```
> print(beta.lasso)
```

```
23 x 1 sparse Matrix of class "dgCMatrix"
                    1
(Intercept) 84.466328
advertising 4.977805
Ind_898    .
Ind_1934   .
Ind_3184   .
Ind_4399   .
Ind_4513   .
Ind_6092   .
Ind_901    .
Ind_5880   .
Ind_19353 .
Ind_19417 .
Ind_25718 .
Ind_25719 .
Ind_25884 .
Ind_25705 .
Ind_25707 .
Ind_25714 .
Ind_25716 .
Ind_25538 .
Ind_25540 .
Ind_25547 .
Ind_25549 .
```

We observe that LASSO was able to identify that only advertising is relevant and remove all other predictors automatically, even when the number of variables was larger than the number of observations. Let us compare this solution with the OLS solution:

```r
# OLS fit
fit.ols <- lm(sales ~ advertising, data)
beta.ols <- coef(fit.ols)

# Compare the two sets of coefficients
beta.lasso <- as.numeric(beta.lasso)[1:2]
beta.diff <- beta.ols - beta.lasso
print(beta.diff)
```

We can see in the console output that the differences are non-zero.

```
> print(beta.diff)
(Intercept) advertising
 -47.730902 3.799475
```

This is because the 'best' λ is not zero, in which case LASSO would be identical to OLS.

```
> fit.cv$lambda.1se
[1] 10.63842
```

Finally let us compare the quality of the fit:

```r
yhat.fit.cv <- predict(fit.cv,as.matrix(data[2:23]))
yhat.fit.ols <- predict(fit.ols,data[2:23])
MSE <- c(mean((data[,1] - yhat.fit.cv)^2), mean((data[,1] - yhat.fit.ols)^2))
names(MSE) <- c("LASSO","OLS")
print(MSE)
```

The resulting MSEs are:

```
> print(MSE)
   LASSO      OLS
207.18950  94.01345
```

Observe that the MSE of LASSO is greater than OLS. This is to be expected, as LASSO attempts to find the optimal balance between bias and variance to avoid overfitting, as controlled by λ. This example also demonstrates how risky is to rely on in-sample fit statistics. Figure 7.7 visualises how the two model fits differ, where it can be seen that the LASSO fit is more resistant to follow the variability observed in the sample, as shrinkage would prescribe.

```r
# Plot the target variable and the two model fits
plot(sales,type="o")
lines(yhat.fit.cv,col="red")
lines(yhat.fit.ols,col="blue")
legend("topright",c("Sales","LASSO","OLS"),col=c("black","red","blue"),lty=1)
```

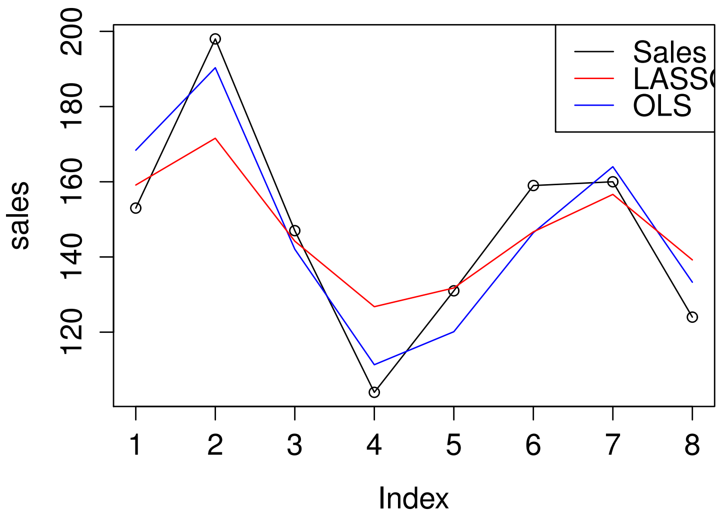


Figure 7.7: LASSO and OLS fit.

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Appendix A. Installing R, RStudio and packages

In order to be able to run the scripts presented in this document, you should have installed the latest versions of the following software:

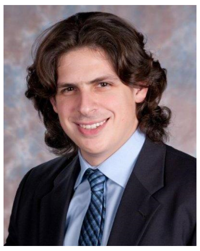
1. **R Statistical Software**, available here.
2. **RStudio Desktop**, available here.
3. Once the above have been installed, you should install (through RStudio) the following **R packages**: `forecast`, `Mcomp`, `tsintermittent`, `hts`, `fpp`, `MAPA`, `lars`, `glmnet`. A short guide on how to install packages through RStudio can be found here.
4. Download and install **RTools**, available here.
5. Install the `TStools` package available on github. You can install this by typing and running the following two commands in the console of RStudio:

   ```
   if (!require("devtools")){install.packages("devtools")}
   devtools::install_github("trnnick/TStools")
   ```

   The package depends on `Rcpp` and `RcppArmadillo`, which will be installed automatically. However Mac OS users may need to install **gfortran** libraries in order to use `Rcpp`. Follow this link for the instructions.

All software are available for free. Please proceed with default settings during the installation of the two software.

Appendix B. About the instructors



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Fotios is an Assistant Professor at Cardiff Business School of Cardiff University, the Forecasting Support Systems Editor of *Foresight* and elected Director of the International Institute of Forecasters. His research expertise lies in behavioural aspects of forecasting and improving the forecasting process, applied in the context of business and supply chain. Together with Nikos, he has co-founded of the Forecasting Society (www.forsoc.net). Little known fact about Fotios: his forecast fu is as good as his kung fu!